#1a. Suppose we have eight people and that we must form a committee of three people. How many committees of three can we select from the eight individuals?

\[
\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 1 \cdot 5!} = \frac{8 \cdot 7 \cdot 6}{6} = 56
\]

#1b. Suppose that the first person selected is the chairman, the second is the vice-chairman, and the last is the secretary. How many such committees of three can we select from the set of eight people?

\[
3! \binom{8}{3} = 3! \frac{8!}{3!(8-3)!} = \frac{8 \cdot 7 \cdot 6!}{5!} = 8 \cdot 7 \cdot 6 = 336
\]

#1c. A farmer buys three cows, two pigs, and four hens from a man who has six cows, five pigs, and eight hens. In how many ways can the farmer buy the three cows, two pigs, and four hens?

\[
\binom{6}{3} \binom{5}{2} \binom{8}{4} = \frac{6!}{3!3!} \frac{5!}{2!2!} \frac{8!}{4!4!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!6} \frac{5 \cdot 4 \cdot 3!}{2 \cdot 3!} \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{24 \cdot 4!} = 20 \cdot 10 \cdot 70 = 14,000
\]
Suppose we toss a fair coin three times.

#2a. State the sample space.

\[ S = \{HHH, \]  
\[ \text{HHT, HTH, THH,} \]  
\[ \text{HTT, THT, TTH,} \]  
\[ \text{TTT}\} \]

#2b. What is the probability of getting exactly two heads?

\[ P(\text{exactly 2 H’s}) = P(HHT) + P(HTH) + P(THH) \]
\[ = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8} \]

#2c. What is the probability of getting exactly two heads or exactly three heads?

\[ P(\text{exactly 2 H’s or exactly 3 H’s}) = P(\text{2 H’s}) + P(\text{3 H’s}) \]
\[ = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2} \]

#2d. What is the probability of getting at most two heads?

\[ P(\text{at most 2 H’s}) = P(\text{no H’s}) + P(\text{exactly 1 H}) + P(\text{exactly 2 H’s}) \]
\[ = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8} \]
\[ = 1 - P(\text{exactly 3 H’s}) = 1 - \frac{1}{8} = \frac{7}{8} \]
Suppose we have a standard deck of 52 cards and that we are going to draw one card from the deck.

#3a. What is the probability of drawing a face card (i.e., a jack, queen or king)?

\[ P(\text{face card}) = \frac{12}{52} \]

#3b. What is the probability of drawing a red card (i.e., a heart or a diamond)?

\[ P(\text{red card}) = \frac{26}{52} \]

#3c. What is the probability of drawing a red face card?

\[ P(\text{red card} \& \text{face card}) = \frac{6}{52} \]
Suppose events A and B have the following probabilities:

\[
P(A) = \frac{6}{10}, \quad P(B) = \frac{3}{10} \quad \text{and} \quad P(A \text{ and } B) = \frac{2}{10}.
\]

#4a. What is the probability that A or B occurs?

\[
P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{6}{10} + \frac{3}{10} - \frac{2}{10} = \frac{7}{10}
\]

#4b. What is the probability that A does not occur?

\[
P(\text{not } A) = 1 - P(A) = 1 - \frac{6}{10} = \frac{4}{10}
\]

#4c. What is the probability of A given B?

\[
P(A/B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{\frac{2}{10}}{\frac{3}{10}} = \frac{2}{3}
\]
#5. Suppose we have a production run of 12 items, of which 4 are defective. Three items are drawn from the 12 without replacement. What is the probability that all three are defective?

\[ P(3 \text{ defective}) = \frac{\binom{4}{3} \cdot \binom{8}{0}}{\binom{12}{3}} = \frac{1}{55} \]

Suppose we have three boxes labeled X, Y, and Z. Box X has ten light bulbs, of which four are defective; Box Y has six light bulbs, of which one is defective; Box Z has eight light bulbs, of which three are defective.

We will select a box at random, and then we will select a light bulb from that box. The light bulb will be either defective (D) or non-defective (N).

#6a. What is the probability that we select box Y and a defective light bulb? (Leave the answer in fractions.)

\[ P(D \& Y) = P(D/Y)P(Y) = \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18} \]

#6b. What is the probability the light bulb came from box Y given that it is defective? (Leave the answer in fractions.)

\[ P(Y/D) = \frac{P(Y \text{ and } D)}{P(D)} = \frac{1}{18} \cdot \frac{2}{15 + \frac{1}{18} + \frac{3}{24}} \]
A fair die is thrown twice.

#7a. What is the probability a 5 appears on the first throw?

S =

\[
\begin{array}{ccccccc}
1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\
2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\
3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\
4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\
5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\
6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6
\end{array}
\]

\[
P(5 \text{ on first throw}) = P(5,1 \text{ or } 5,2 \text{ or } 5,3 \text{ or } 5,4 \text{ or } 5,6) = \frac{6}{36} = \frac{1}{6}
\]

#7b. What is the probability the sum on the two dice is greater than or equal to 10 given that the first throw was a 5?

\[
P(\text{sum} \geq 10/5 \text{ on first throw}) = \frac{P(\text{sum} \geq 10 \text{ and } 5 \text{ on first})}{P(5 \text{ on first})} = \frac{P(5,5) + P(5,6)}{\frac{6}{36} + \frac{6}{36}} = \frac{\frac{1}{36} + \frac{1}{36}}{\frac{6}{36} + \frac{6}{36}} = \frac{2}{6}
\]

or

note that 5,5 and 5,6 are two elements of the six elements in the 5-row.
#8. Suppose you face a decision involving a gamble that pays 100 with probability .25 and -10 with probability .75. What is the expected value of this gamble?

<table>
<thead>
<tr>
<th>X</th>
<th>100</th>
<th>-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>.25</td>
<td>.75</td>
</tr>
</tbody>
</table>

\[ E[X] = .25(100) + .75(-10) = 25 - 7.50 = 17.50 \]

Suppose \( x_b \) is a binomial random variable with \( n = 6 \) and \( p = \frac{1}{2} \).

#9a. What is the mean of \( x_b \), i.e., what is \( E(x_b) \)?

\[ E[x_b] = np = (6)(.5) = 3 \]

#9b. What is the variance of \( x_b \), i.e., what is \( V(x_b) \)?

\[ V[x_b] = np(1-p) = (6)(.5)(.5) = 1.5 \]

#9c. Find the probability that \( x_b \) equals 2.

\[ P(x_b = 2/n = 6, p = 1/2) = \binom{6}{2} \left( \frac{1}{2} \right)^2 \left( \frac{1}{2} \right)^4 = 15 \cdot \frac{1}{4} \cdot \frac{1}{16} = \frac{15}{64} \]
Suppose $x_N$ is a normal random variable with a mean (i.e., $\mu$) of 2 and a variance (i.e., $\sigma^2$) of 16 (and a standard deviation of 2).

#10a. Find $P(2 \leq x \leq 6)$

$$P(2 \leq x \leq 6) = P\left(\frac{2 - 2}{\frac{\sigma}{\sqrt{N}}} \leq z \leq \frac{6 - 2}{\frac{\sigma}{\sqrt{N}}}\right) = P\left(0 \leq z \leq \frac{4}{\frac{\sigma}{\sqrt{N}}}\right) = P(0 \leq z \leq 1)$$

$$= P(-\infty \leq z \leq 1) - P(-\infty \leq z \leq 0) = .8413 - .5000 = .3413$$

#10b. Find $P(-2 \leq x \leq 6)$

$$P(-2 \leq x \leq 6) = P\left(\frac{-2 - 2}{\frac{\sigma}{\sqrt{N}}} \leq z \leq \frac{6 - 2}{\frac{\sigma}{\sqrt{N}}}\right) = P\left(\frac{-4}{\frac{\sigma}{\sqrt{N}}} \leq z \leq \frac{4}{\frac{\sigma}{\sqrt{N}}}\right) = P(-1 \leq z \leq 1)$$

$$= P(-\infty \leq z \leq 1) - P(-\infty \leq z \leq -1) = .8413 - .1587 = .6826$$

#10c. Find $P(-4 \leq x \leq -2)$

$$P(-4 \leq x \leq -2) = P\left(\frac{-4 - 2}{\frac{\sigma}{\sqrt{N}}} \leq z \leq \frac{-2 - 2}{\frac{\sigma}{\sqrt{N}}}\right) = P\left(\frac{-6}{\frac{\sigma}{\sqrt{N}}} \leq z \leq \frac{-4}{\frac{\sigma}{\sqrt{N}}}\right) = P(-1.5 \leq z \leq -1)$$

$$= P(-\infty \leq z \leq -1) - P(-\infty \leq z \leq -1.5) = .1587 - .0668 = .0919$$