The ‘P’ in Poisson is always capitalized – the distribution is named after its founder, a French mathematician named Poisson. The Poisson distribution has the probability mass function

\[ f(x) = \frac{e^{-\lambda} \lambda^x}{x!} \text{ for } x = 0, 1, 2, \ldots \]

otherwise

Note that the random variable in the Poisson is a countably infinite discrete random variable. The Poisson distribution has the somewhat odd property that \( E[x] = \lambda \) and \( \text{VAR}[x] = \lambda \).

The Poisson distribution will be used in IBC in simulating queuing systems and production systems. Specifically, the Poisson models arrivals. The text example (page 90) is quite good. Suppose the average number of customers arriving at an ATM during the lunch hour is \( \lambda = 12 \) customers per hour. The probability that exactly 5 customers arrive during the lunch hour is

\[ f(x) = \frac{e^{-12} 12^5}{5!} = 0.01274064 . \]

The numerical value is best found using Excel’s POISSON function. Note that the POISSON function has a third term. To find \( P(x= 5) \) we enter =POISSON(5,12,FALSE). The third term, here FALSE, tells Excel to generate the probability mass function.

If we want to find \( P(x > 30) \), then we need the cumulative function. This is found in Excel by entering TRUE instead of FALSE. Specifically, if we enter =POISSON(30,12,TRUE), then Excel reports 0.99999628352266. This is the value of \( P(x \leq 30) \). Thus, \( P(x > 30) = 1 - P(x \leq 30) = 1 - 0.99999628352266 = 0.00000337164773400733 \), which is less than 0.000005 as noted in the text.

How might this information be used? Suppose the average number of customers during the lunch hour is 12. The lunch hour is a peak load period (i.e., a period of high demand). Suppose the average customer needs 5 minutes to conduct their business. Then the ATM can serve exactly the 12 customers seen on average. As such, presuming the customers arrive every 5 minutes, no customer would have to wait.

Note, however, that the ATM is right on the edge of its capacity. If 30 customers arrive during the lunch hour, then many of these customers will be in line or at the ATM machine long after lunch hour is over. As we have seen, the probability of 30 or more customers is virtually zero.
In IBC you will encounter queuing theory. This theory provides the analytical tools needed to determine waiting time and line length in a service system, and thereby the tools needed to determine the proper number of servers (here ATMs). This kind of analysis is particularly relevant to the design of physical systems (e.g., banks, airports, etc.) and the scheduling of workers (e.g., tellers, ticket agents, etc.)

For example, food retailers, like Safeway and Rosaur, typically allow line lengths to be at most 3 customers. If a waiting line exceeds 3 customers, then the retailer opens another check-out line. The manager of the store must have an additional worker on duty in order to open the additional line. This kind of scheduling problem exists in all retail situations, and thereby constitutes a common HRM problem. Note that the manager can open another line only if the physical equipment (i.e., counter, cash register, etc.) for the line exists. This constitutes a design problem for the architects.

Queuing theory typically assumes that the arrival of customers is described by the Poisson distribution, and that the time required to serve a customer is described by the exponential distribution. Thus, the Poisson and exponential distributions are particularly important.

PROBLEM:
Ch. 3, #20