DEFINITION: $P(X/Y) = \frac{P(X \cap Y)}{P(Y)}$

$P(X/Y)$ is called the conditional probability of $X$ given $Y$, and is pronounced “the probability of $X$ given $Y$”.

**SPECIAL MULTIPLICATION RULE:**

if $P(X/Y) = P(X)$, then $P(X \cap Y) = P(X)P(Y)$

DEFINITION: $X$ and $Y$ are independent if and only if $P(X/Y) = P(X)$.

**GENERAL MULTIPLICATION RULE:**

if $P(X/Y) \neq P(X)$, then $P(X \cap Y) = P(X/Y)P(Y) = P(Y/X)P(X)$

For example, suppose:

$P(X) = .6$
$P(Y) = .5$
$P(X \cap Y) = .2$

Then, by the definition of conditional probability,

$P(X/Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{.2}{.5} = .4$

$P(Y/X) = \frac{P(X \cap Y)}{P(X)} = \frac{.2}{.6} = .33\overline{3}$

Are $X$ and $Y$ independent? To answer this question, we must determine if $P(X/Y) = P(X)$ is true.

We were given $P(X) = .6$ and we calculated $P(X/Y) = .4$. Since $.4 \neq .6$, we conclude that $X$ and $Y$ are not independent.

Note that independence and dependence are symmetric relations. That is, if $X$ and $Y$ are (or are not) independent, then $Y$ and $X$ are (or are not) dependent. For example, we have $P(Y) = .5$ and $P(Y/X) = .33\overline{3}$. Since $.5 \neq .33\overline{3}$, $Y$ and $X$ are not independent.

By the general multiplication rule, we have

$P(X \cap Y) = P(X/Y)P(Y) = (.4)(.5) = .2$

$= P(Y/X)P(X) = (.33\overline{3})(.6) = .2$
PROBLEM:
Suppose events A and B have the following probabilities:
P(A) = .6, P(B) = .3 and P(A∩B) = .2.

C. What is the probability of A given B?
D. What is the probability of B given A?