Probability theory is characterized by three axioms. A probability system is a triple $<S,S^*,P>$ where $S$ is a sample space, $S^*$ is a sigma algebra on $S$, and $P$ is measure from $S^*$ to the real numbers. The axioms are due to Kolmogorov (1933) and are stated in set-theoretic terms as follows:

**AXIOMS**

Axiom #1: $P(X) \geq 0$ for all events $X$ in $S^*$

Axiom #2: $P(S) = 1$

Axiom #3: if $X \cap Y = \emptyset$, then $P(X \cup Y) = P(X) + P(Y)$ for all events $X$ and $Y$ in $S^*$

where $\cap$ denotes intersection (i.e., “and”), $\cup$ denotes union (i.e., “or”), and $\emptyset$ denotes the empty set.

The first two axioms state that probabilities are numbers between 0 and 1 (Rules 1 and 2 in the text), and the third axiom states that if two events are mutually exclusive (i.e., jointly impossible), then the probability of the disjunction of the events is the sum of the probabilities (Rule 4 in the text).

**SPECIAL ADDITION RULE (Axiom #3):**

if $X \cap Y = \emptyset$, then $P(X \cup Y) = P(X) + P(Y)$

For example, suppose:

$P(X) = .3$

$P(Y) = .5$

$X \cap Y = \emptyset$

Then, by the special addition rule,

$P(X \cup Y) = P(X) + P(Y) = .3 + .5 = .8$
**GENERAL ADDITION RULE:**

if $X \cap Y \neq \emptyset$, then $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

For example, suppose:

$P(X) = .6$

$P(Y) = .5$

$P(X \cap Y) = .2$  (NOTE: if $P(X \cap Y) > 0$, then $X \cap Y \neq \emptyset$.)

Then, by the general addition rule,

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = .6 + .5 -.2 = .9$$

Graphically, we have the following:

If the area of $S^*$ is 1, then we can take $P(X)$ to be the area of $X$, $P(Y)$ to be the area of $Y$, and $P(X \cap Y)$ to be the area of $X \cap Y$. Then $P(X)+P(Y)$ counts the area of $X \cap Y$ twice, and we must subtract it out once. Thus, when $X \cap Y \neq \emptyset$, the addition rule is $P(X)+P(Y)-P(X \cap Y)$.

Note that the special additional rule is just a special case of the general addition rule. Graphically, if $X \cap Y = \emptyset$, then we have the following:

If $X \cap Y = \emptyset$, then $P(X \cap Y) = P(\emptyset) = 0$, so the general additional rule becomes the special addition rule, as follows

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = P(X) + P(Y) - 0 = P(X) + P(Y).$$
PROBLEM:
Suppose events A and B have the following probabilities:
P(A) = .6, P(B) = .3 and P(A \cap B) = .2.

A. What is the probability that A or B (i.e., A \cup B) occurs?
B. What is the probability that A does not occur?