Math 107
Final Exam
Fall 1998

You must show your work in order to receive credit; no credit for guessing. Calculators may be used but are not necessary.
Total points: 200     Total pages: 7

1. (1 pt)  F  (Circle one). I want one free point.

2. (6 pts) Multiple Choice: Circle the one correct answer in each part.
   a. What is the domain of the function \( f(x) = (x + 3)(x - 1) \)?
      all \( x \) except \( x = -3 \)       all \( x \) except \( x = 1 \) and \( x = -3 \)
      all \( x \) except \( x = 1 \)  \(\text{all } x\)
   b. What is the domain of the function \( g(x) = \frac{-3}{x - 1} \)?
      all \( x \) except \( x = -3 \)       all \( x \) except \( x = 1 \) and \( x = 3 \)
      all \( x \) except \( x = 1 \)  \(\text{all } x\)

3. (4 pts) Sketch the graph of the function \( f(x) = 2x + 3 \).

   \[
   \begin{align*}
   y &= mx + b \\
   b &= y \text{ int} \\
   m &= \text{slope} \\
   
   \end{align*}
   \]

   \( b = 3 \) \( m = 2 \)

   (3 pts) Label (with coordinates) 3 points on the graph.

   (3 pts) What is the \( x \)-intercept?  \(-1.5\)

   \[
   x \text{-int } y = 0
   \]

   \[
   \begin{align*}
   0 &= 2x + 3 \\
   -3 &= 2x \\
   x &= -\frac{3}{2}
   \end{align*}
   \]
4. (3 pts) Let \( f(x) = x^2 - 2x + 3 \). What is the value of \( f(2) \)?
\[
f(x) = (2)^2 - 2(2) + 3 = 4 - 4 + 3 = 3
\]
\( f(2) = 3 \)

5. (3 pts) Let \( f(x) = x^2 - 2x + 3 \). What is the value of \( f(-3) \)?
\[
f(-3) = (-3)^2 - 2(-3) + 3 = 9 + 6 + 3 = 18
\]
\( f(-3) = 18 \)

6. (6 pts ea) Given the points \( P(-3, 2) \) and \( Q(-4, -1) \), answer the following questions.

a. Find the distance between \( P \) and \( Q \). (by Pythagorean Theorem):
\[
d = \sqrt{[-3 - (-4)]^2 + [2 - (-1)]^2} = \sqrt{1 + 9} = \sqrt{10}
\]

b. What is the slope of the line passing through these points?
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{-4 - (-3)} = \frac{-3}{-1} = 3
\]
\( m = 3 \)

c. Find the equation of the line through these points.
\[
y - y_1 = m(x - x_1)
\]
\[
y + 1 = 3(x + 4)
\]
\[
y + 1 = 3x + 12
\]
\[
y = 3x + 11
\]
(standard form)

7. a. (6 pts) Solve for \( x \):
\[
3(x + 1) - 5(x + 2) \leq 1
\]
\[
3x + 3 - 5x - 10 \leq 1
\]
\[
-2x - 7 \leq 1
\]
\[
-2x \leq 8
\]
\[
x \geq -4
\]

b. (2 pts) Express the solution in interval notation: \([-4, \infty)\)
8. (4 pts) Solve for $x$: $|x - 2| < 3$.

$$\begin{align*}
-3 &< x - 2 < 3 \\
-1 &< x < 5
\end{align*}$$

9. (5 pts) Solve for $x$: $6(x - 1) - 2(2x + 1) = 3$.

$$\begin{align*}
6x - 6 - 4x - 2 &= 3 \\
2x &= 11 \\
x &= \frac{11}{2}
\end{align*}$$

10. (6 pts) Solve for $x$: $\frac{x + 3}{2} + \frac{x - 2}{3} = 2$.

$$\begin{align*}
\frac{3(x + 3) + 2(x - 2)}{6} &= \frac{12}{6} \\
3x + 9 + 2x - 4 &= 12 \\
5x &= 7 \\
x &= \frac{7}{5}
\end{align*}$$

11. (6 pts) Solve for $a$: $A = \frac{1}{2}(a + b)h$.

$$\begin{align*}
2 \left[ A + \frac{1}{2}(a+b)h \right] &= 2A \\
\frac{2A}{h} &= (a+b)
\end{align*}$$

12. (6 pts) Solve for $z$: $z^2 - 2z + 5 = 0$.

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)} = \frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$
13. (6 pts) Solve for $x$:

$$\sqrt{(x+1)^2} = \sqrt{3}$$

$x+1 = \sqrt{3}$

$$x = -1 + \sqrt{3}$$

14. (6 pts) Solve for $x$:

$$2x^2 - 5x - 12 = 0$$

$$(2x + 3)(x - 4) = 0$$

$x-4 = 0$

$x = 4$

$x+3 = 0$

$x = -3$

$$x = \frac{-3}{2}$$

15. (6 pts) Solve for $x$ by completing the square:

$$x^2 - 6x - 7 = 0$$

$$x^2 - 6x + 9 = 9$$

$$(x - 3)^2 = 16$$

$$x - 3 = \pm 4$$

$$x = -1 \quad \text{or} \quad x = 7$$

16. (8 pts) Solve for $y$:

$$y - 3 = 2\sqrt{y}$$

$$(y - 3)^2 = 4y$$

$$y^2 - 6y + 9 = 4y$$

$$y^2 - 10y + 9 = 0$$

$$(y - 1)(y - 9) = 0$$

$$y = 1 \quad \text{or} \quad y = 9$$

17. (8 pts) Simplify:

$$\frac{x^2 + 2x - 3}{2 - x - x^2} = \frac{x^2 + 2x - 1}{x^2 + x - 2}$$

$$\frac{(x+3)(x-1)}{(x-1)(x+2)} = \frac{-x+3}{x+2}$$
\[
\sqrt{18.} \ (8 \text{ pts}) \text{ Simplify:} \quad \frac{x+2}{x^2+x} - \frac{1}{x} = \frac{x+2 - \frac{x+1}{x}}{x(x+1)} = \frac{x}{x(x+1)} \\
\text{LCM:} \quad \frac{x}{x+1} \left[ \frac{x+2}{x(x+1)} - \frac{1}{x} \right] \\
\frac{x^2 - x - 12}{x^2 + x} = (x-4)(x+3)
\]

\[
\sqrt{19.} \ (8 \text{ pts}) \text{ Solve for } x: \\
\frac{1}{x-4} - \frac{2}{x+3} = \frac{3}{x^2 - x - 12} \\
(x+3) - 2(x-8) = 3 \quad \Rightarrow \quad -x = -8 \\
x = 8
\]

\[
\sqrt{20.} \ (4 \text{ pts}) \text{ Simplify:} \\
\sqrt[3]{2}\sqrt[3]{x^2} \cdot \sqrt[3]{y^3} = 3 \cdot \sqrt[6]{2} \cdot x \cdot \sqrt[6]{y^2} \cdot \sqrt[6]{y} \\
\sqrt[3]{9} \cdot \sqrt[3]{8} \cdot x \cdot \sqrt[3]{y^2} \cdot \sqrt[3]{y} \\
\sqrt[3]{3} \cdot \sqrt[3]{4} \cdot x \cdot \sqrt[3]{y^2} \cdot \sqrt[3]{y} \\
\sqrt[3]{3} \cdot \sqrt[3]{4} \cdot x \cdot y \cdot \sqrt[3]{y}
\]

\[
\sqrt{21.} \ (6 \text{ pts}) \text{ Rationalize the denominator and simplify:} \\
\frac{1}{\sqrt{x-1}} \cdot \frac{\sqrt{x+1}}{\sqrt{x+1}} = \frac{\sqrt{x+1}}{x-1}
\]

\[
\sqrt{22.} \ (5 \text{ pts ea}) \text{ Simplify and express your results using positive exponents:} \\
\sqrt{-2} \cdot \left(\frac{-36a^2b^2}{8a-1b^2} \right)^{-2} = \frac{4}{9} \cdot \frac{a^2}{b^2} \\
\left(\frac{-9a^3}{b^2} \right)^{-2} = \frac{b^4}{16a^6}
\]

\text{RESERVED} \\
\text{Pre-Vet} \quad \text{Van Derwel}
b. \( (2x^{\frac{1}{2}})(3x^{-\frac{1}{2}}) \)
\[ 6x^{\left(\frac{1}{2} - \frac{1}{2}\right)} = 6^{x^0} \]
\[ 6x^0 = 6^{x^0} \]

c. \( (8y^6x^{-3})^{\frac{1}{2}} \rightarrow 8^{\frac{1}{2}} \cdot y^{\frac{6}{2}} \cdot x^{-\frac{3}{2}} = 2 \cdot y^3 \cdot x^{-\frac{3}{2}} = \frac{2y^3}{x^{\frac{3}{2}}} \)

23. (5 pts ea) Factor completely:

\( \sqrt{a. \ 6a^2b^2 - 10a^2b^2 + 2a^2b} \)
\[ 2a^2b(3ab - 5b + 1) \]

\( \checkmark b. \ x^2 + 2x - 15 \)
\[ \sqrt{(x+5)(x-3)} \]

\( \checkmark c. \ xy - 3y + x - 3 \)
\[ \sqrt{(x-3)(y-1)+1(x-3)} \]
\[ (y+1)(x-3) \]

\( \checkmark d. \ 5y^2 - 20y \)
\[ \sqrt{5y(y^2 - 4)} \]
\[ 5y(y+2)(y-2) \]

24. (5 pts) Find the quotient and simplify:
\[ \frac{2i}{4 + 3i} \]
\[ \frac{2i}{4 + 3i} \cdot \frac{4 - 3i}{4 - 3i} \]
\[ \frac{8i - 6i^2}{16 - 9i^2} \]
\[ \frac{8i + 6}{25} \]
25. (8 pts) The sides of a rectangle are 8 feet long and six feet long. Find the length of the diagonals.

\[ d^2 + 8^2 = d^2 \]
\[ d = \sqrt{36 + 64} \]
\[ d = \sqrt{100} \]
\[ d = 10 \]

26. (8 pts) It takes Henry 4 hours to do the dishes, but Mary can do them in 3 hours. How long will it take them to do the dishes together?

<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Henry</td>
<td>4</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>Mary</td>
<td>3</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>together</td>
<td>1</td>
<td>( \frac{1}{t} )</td>
</tr>
</tbody>
</table>

\[ t = \text{time to do the whole job, Q} \]
\[ \frac{1}{4} + \frac{1}{3} = \frac{1}{t} \]
\[ 3t + 4t = 12 \]
\[ 7t = 12 \]
\[ t = \frac{12}{7} \]
\[ t = 1 \frac{5}{7} \text{ hours} \]

27. (8 pts) Find two consecutive positive numbers the sum of whose squares is 85.

First number = \( x \)
2nd number = \( x + 1 \)

\[ x^2 + (x+1)^2 = 85 \]
\[ x^2 + x^2 + 2x + 1 = 85 \]
\[ 2x^2 + 2x + 84 = 0 \]
\[ x^2 + x - 42 = 0 \]

\[ (x+7)(x-6) = 0 \]
\[ x = -7, x = 6 \]
\[ x+1 = 7 \]