HW04, Chapter 26: 2, 5, 12, 18, 21, 27, 31, 33, 45, and 49.

Due Friday, September 30, 2005.

2. Two conductors having net charges of +10.0 µC and –10.0 µC have a potential difference of 10.0 V between them. (a) Determine the capacitance of the system. (b) What is the potential difference between the two conductors if the charges on each are increased to +100 µC and –100 µC?

26.2 (a) \( C = \frac{Q}{\Delta V} = \frac{10.0 \times 10^{-6} \text{ C}}{10.0 \text{ V}} = 1.00 \times 10^{-6} \text{ F} = \boxed{1.00 \mu \text{F}} \)

(b) \( \Delta V = \frac{Q}{C} = \frac{100 \times 10^{-6} \text{ C}}{1.00 \times 10^{-6} \text{ F}} = \boxed{100 \text{ V}} \)

5. Two conducting spheres with diameters of 0.400 m and 1.00 m are separated by a distance that is large compared with the diameters. The spheres are connected by a thin wire and are charged to 7.00 µC. (a) How is this total charge shared between the spheres? (Ignore any charge on the wire.) (b) What is the potential of the system of spheres when the reference potential is taken to be \( V = 0 \) at \( r = \infty \)?

26.5 (a) \( \frac{Q_1}{Q_2} = \frac{R_1}{R_2} \)

\( Q_1 + Q_2 = \left(1 + \frac{R_1}{R_2}\right)Q_2 = 3.50Q_2 = 7.00 \mu \text{C} \)

\( Q_2 = 2.00 \mu \text{C} \quad Q_1 = 5.00 \mu \text{C} \)

(b) \( V_1 = V_2 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{5.00 \mu \text{C}}{\left(8.99 \times 10^9 \text{ m} / \text{F}\right)^{-1} \left(0.500 \text{ m}\right)} = 8.99 \times 10^4 \text{ V} = \boxed{89.9 \text{ kV}} \)

12. A 20.0-µF spherical capacitor is composed of two concentric metal spheres, one having a radius twice as large as the other. The region between the spheres is a vacuum. Determine the volume of this region.

26.12 Let the radii be \( b \) and \( a \) with \( b = 2a \). Put charge \( Q \) on the inner conductor and \( -Q \) on the outer. Electric field exists only in the volume between them. The potential of the inner sphere is \( V_a = k\frac{Q}{a} \), that of the outer is \( V_b = k\frac{Q}{b} \). Then
\[ V_a - V_b = \frac{kQ}{a} - \frac{kQ}{b} = \frac{Q}{4\pi \varepsilon_0} \left( \frac{b-a}{ab} \right) \] and \[ C = \frac{Q}{V_a - V_b} = \frac{4\pi \varepsilon_0}{b-a} \cdot \]

Here \[ C = \frac{4\pi \varepsilon_0}{a} \Rightarrow 2\frac{a}{2} = 8\pi \varepsilon_0 \cdot \]

The intervening volume is

\[
\text{Volume} = \frac{4}{3} \pi b^3 - \frac{4}{3} \pi a^3 = \frac{7}{3} \left( \frac{4}{3} \pi \right) a^3 = \frac{7C^3}{8\pi^2 \varepsilon_0^2}
\]

The outer sphere is 360 km in diameter.

18. Evaluate the equivalent capacitance of the configuration shown in Figure P26.18. All the capacitors are identical, and each has capacitance \( C \).

\[ C_{eq} = C \left( 1 + \frac{1}{2} + \frac{1}{3} \right) = \frac{11}{6} C = 1.83C \]

P26.18 The circuit reduces first according to the rule for capacitors in series, as shown in the figure, then according to the rule for capacitors in parallel, shown below.

21. Four capacitors are connected as shown in Figure P26.21. (a) Find the equivalent capacitance between points \( a \) and \( b \). (b) Calculate the charge on each capacitor if \( \Delta V_{ab} = 15.0 \text{ V} \).
27. Find the equivalent capacitance between points a and b for the group of capacitors connected as shown in Figure P26.27. Take \( C_1 = 5.00 \mu F \), \( C_2 = 10.0 \mu F \), and \( C_3 = 2.00 \mu F \).
31. (a) A 3.00-μF capacitor is connected to a 12.0-V battery. How much energy is stored in the capacitor? (b) If the capacitor had been connected to a 6.00-V battery, how much energy would have been stored?

P26.31

(a) \[ U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} (3.00 \, \mu F)(12.0 \, V)^2 = 216 \, \mu J \]

(b) \[ U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} (3.00 \, \mu F)(6.00 \, V)^2 = 54.0 \, \mu J \]

33. Two capacitors, \( C_1 = 25.0 \, \mu F \) and \( C_2 = 5.00 \, \mu F \), are connected in parallel and charged with a 100-V power supply. (a) Draw a circuit diagram and calculate the total energy stored in the two capacitors. (b) What If? What potential difference would be required across the same two capacitors connected in series in order that the combination stores the same amount of energy as in (a)? Draw a circuit diagram of this circuit.

P26.33

\[ U = \frac{1}{2} C (\Delta V)^2 \]

The circuit diagram is shown at the right.

(a) \[ C_p = C_1 + C_2 = 25.0 \, \mu F + 5.00 \, \mu F = 30.0 \, \mu F \]
\[ U = \frac{1}{2} (30.0 \times 10^{-6})(100)^2 = 0.150 \, J \]

(b) \[ C_s = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left( \frac{1}{25.0 \, \mu F} + \frac{1}{5.00 \, \mu F} \right)^{-1} = 4.17 \, \mu F \]
\[ U = \frac{1}{2} C (\Delta V)^2 \]
\[ \Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(0.150)}{4.17 \times 10^{-6}}} = 268 \, V \]
45. A commercial capacitor is to be constructed as shown in Figure 26.17a. This particular capacitor is made from two strips of aluminum separated by a strip of paraffin-coated paper. Each strip of foil and paper is 7.00 cm wide. The foil is 0.004 00 mm thick, and the paper is 0.025 0 mm thick and has a dielectric constant of 3.70. What length should the strips have, if a capacitance of $9.50 \times 10^{-8}$ F is desired before the capacitor is rolled up? (Adding a second strip of paper and rolling the capacitor effectively doubles its capacitance, by allowing charge storage on both sides of each strip of foil.)

\[
P_{26.45} \quad C = \frac{\kappa \varepsilon_0 A}{d}
\]

or

\[
95.0 \times 10^{-3} = \frac{3.70 \left(8.85 \times 10^{-12}\right) \left(0.0700\right) \ell}{0.0250 \times 10^{-3}}
\]

\[
\ell = 1.04 \text{ m}
\]

49. Each capacitor in the combination shown in Figure P26.49 has a breakdown voltage of 15.0 V. What is the breakdown voltage of the combination?

![Figure P26.49](image)

\[\text{P26.49} \quad \text{The given combination of capacitors is equivalent to the circuit diagram shown to the right.}
\]

Put charge $Q$ on point $A$. Then,

\[
Q = (40 \mu F) \Delta V_{AB} = (10 \mu F) \Delta V_{BC} = (40 \mu F) \Delta V_{CD}.
\]

So, $\Delta V_{BC} = 4\Delta V_{AB} = 4\Delta V_{CD}$, and the center capacitor will break down first, at $\Delta V_{BC} = 15.0 \text{ V}$. When this occurs,

\[
\Delta V_{AB} = \Delta V_{CD} = \frac{1}{4}(\Delta V_{BC}) = 3.75 \text{ V}
\]

and

\[
V_{AD} = V_{AB} + V_{BC} + V_{CD} = 3.75 \text{ V} + 15.0 \text{ V} + 3.75 \text{ V} = 22.5 \text{ V}.
\]