1. An electric field with a magnitude of 3.50 kN/C is applied along the x axis. Calculate the electric flux through a rectangular plane 0.350 m wide and 0.700 m long assuming that (a) the plane is parallel to the yz plane; (b) the plane is parallel to the xy plane; (c) the plane contains the y axis, and its normal makes an angle of 40.0° with the x axis.

\[ \Phi = EA \cos \theta = (3.50 \times 10^3)(0.350 \times 0.700) \cos 0^\circ = 858 \text{ N} \cdot \text{m}^2/\text{C} \]

(b) \[ \theta = 90^\circ \quad \Phi = 0 \]

(c) \[ \Phi = (3.50 \times 10^3)(0.350 \times 0.700) \cos 40.0^\circ = 657 \text{ N} \cdot \text{m}^2/\text{C} \]

4. Consider a closed triangular box resting within a horizontal electric field of magnitude \( E = 7.80 \times 10^4 \) N/C as shown in Figure P24.4. Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the box.

\[ \Phi_{E, A'} = EA' \cos \theta = (7.80 \times 10^4)(0.030 \times 0.030) \cos 180^\circ = -2.34 \text{ kN} \cdot \text{m}^2/\text{C} \]

(b) \[ \Phi_{E, A} = EA \cos \theta = (7.80 \times 10^4)(A) \cos 60^\circ \]
\[
A = \left(30.0 \text{ cm}\right) \left(10.0 \text{ cm}\right) \frac{1}{\cos 60^\circ} = 600 \text{ cm}^2 = 0.0600 \text{ m}^2
\]
\[
\Phi_{E, A} = \left(7.80 \times 10^4\right) \left(0.0600 \text{ m}\right) \cos 60^\circ = +2.34 \text{ kN} \cdot \text{m}^2/\text{C}
\]

(c) The bottom and the two triangular sides all lie parallel to \(E\), so \(\Phi_E = 0\) for each of these. Thus,
\[
\Phi_{E, \text{total}} = -2.34 \text{ kN} \cdot \text{m}^2/\text{C} + 2.34 \text{ kN} \cdot \text{m}^2/\text{C} + 0 + 0 + 0 = 0
\]

9. The following charges are located inside a submarine: 5.00 \(\mu\text{C}\), –9.00 \(\mu\text{C}\), 27.0 \(\mu\text{C}\), and –84.0 \(\mu\text{C}\). (a) Calculate the net electric flux through the hull of the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

\[
\Phi_E = \frac{q}{\varepsilon_0} = \frac{\left(+5.00 \mu\text{C} - 9.00 \mu\text{C} + 27.0 \mu\text{C} - 84.0 \mu\text{C}\right)}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = -6.89 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}^2
\]
\[
\Phi_E = -6.89 \text{ MN} \cdot \text{m}^2/\text{C}
\]

(b) Since the net electric flux is negative, more lines enter than leave the surface.

11. Four closed surfaces, \(S_1\) through \(S_4\), together with the charges \(-2Q\), \(Q\), and \(-Q\) are sketched in Figure P24.11. (The colored lines are the intersections of the surfaces with the page.) Find the electric flux through each surface.

**Figure P24.11**

\[
\Phi_E = \frac{q}{\varepsilon_0}
\]

Through \(S_1\) \[
\Phi_E = \frac{-2Q + Q}{\varepsilon_0} = \frac{-Q}{\varepsilon_0}
\]

Through \(S_2\) \[
\Phi_E = \frac{+Q - Q}{\varepsilon_0} = 0
\]
Through $S_3$ \[ \Phi = \Phi_3 = \frac{-2Q + Q - Q}{\varepsilon_0} = \frac{-2Q}{\varepsilon_0} \]

Through $S_4$ \[ \Phi = \Phi_4 = 0 \]

16. In the air over a particular region at an altitude of 500 m above the ground the electric field is 120 N/C directed downward. At 600 m above the ground the electric field is 100 N/C downward. What is the average volume charge density in the layer of air between these two elevations? Is it positive or negative?

P24.16 Consider as a gaussian surface a box with horizontal area $A$, lying between 500 and 600 m elevation.

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} = (120 \text{ N/C})A - (100 \text{ N/C})A = \frac{\rho A(100 \text{ m})}{\varepsilon_0} \]

\[ \rho = \frac{(20 \text{ N/C})(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)}{100 \text{ m}} = 1.77 \times 10^{-12} \text{ C/m}^3 \]

The charge is positive, to produce the net outward flux of electric field.

19. An infinitely long line charge having a uniform charge per unit length $\lambda$ lies a distance $d$ from point $O$ as shown in Figure P24.19. Determine the total electric flux through the surface of a sphere of radius $R$ centered at $O$ resulting from this line charge. Consider both cases, where $R < d$ and $R > d$.

![Figure P24.19](image)

P24.19 If $R \leq d$, the sphere encloses no charge and $\Phi = \Phi_0 = \frac{q_0}{\varepsilon_0} = 0$.

If $R > d$, the length of line falling within the sphere is $2\sqrt{R^2 - d^2}$
so $\Phi_s = \frac{2\lambda \sqrt{R^2 - d^2}}{\varepsilon_0}$

24. A solid sphere of radius 40.0 cm has a total positive charge of 26.0 $\mu$C uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.

\[ P24.24 \]
\[
\begin{align*}
(a) & \quad E = \frac{kQr}{a^2} = 0 \\
(b) & \quad E = \frac{kQr}{a^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})(0.100)}{(0.400)^3} = 365 \text{ kN/C} \\
(c) & \quad E = \frac{kQr^2}{a^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.400)^2} = 1.46 \text{ MN/C} \\
(d) & \quad E = \frac{kQr^2}{a^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.600)^2} = 649 \text{ kN/C} \\
\end{align*}
\]

The direction for each electric field is radially outward.

30. A solid plastic sphere of radius 10.0 cm has charge with uniform density throughout its volume. The electric field 5.00 cm from the center is 86.0 kN/C radially inward. Find the magnitude of the electric field 15.0 cm from the center.

\[ P24.30 \]
\[
\begin{align*}
E_1 \cdot 4\pi r_1^2 &= \frac{q_{\text{inside}}}{\varepsilon_0} = \frac{4\pi r_1^2 \rho}{3 \varepsilon_0} \quad E_1 = \frac{\rho}{3 \varepsilon_0} \\
\rho &= \frac{3 \varepsilon_0 E_1}{r_1^2} = \frac{3(8.85 \times 10^{-12} \text{ C}^2)(-86 \times 10^3 \text{ N})}{0.05 \text{ m}^2 \text{ N m}^{-2} \text{ C}^{-1}} = -4.57 \times 10^{-5} \text{ C m}^{-3}.
\end{align*}
\]

Now for the field outside at $r_3 = 15$ cm
\[
E_3 \cdot 4\pi r_3^2 = -\frac{4\pi r_3^2 \rho}{3 \varepsilon_0} \quad E_3 = \frac{\rho}{3 \varepsilon_0} \\
E_3 = \frac{4\pi}{r_3} \cdot \frac{0.10 \text{ m}^3}{3} \cdot (-4.57 \times 10^{-5} \text{ C} \text{ m}^{-3}) = \frac{8.99 \times 10^3 \text{ N m}^2 \text{ C}^{-1}}{(0.15 \text{ m})^2 \text{ C}^2} = -7.64 \times 10^4 \text{ N/C}
\]

36. An insulating sphere is 8.00 cm in diameter and carries a 5.70-µC charge uniformly distributed throughout its interior volume. Calculate the charge
enclosed by a concentric spherical surface with radius (a) \( r = 2.00 \text{ cm} \) and (b) \( r = 6.00 \text{ cm} \).

**P24.36**  
(a) \[
\rho = \frac{Q}{\frac{4}{3} \pi a^3} = \frac{5.70 \times 10^{-6}}{\frac{4}{3} \pi (0.0400 \text{ cm})^3} = 2.13 \times 10^{-2} \text{ C/m}^3
\]

\[
q_m = \rho \left( \frac{4}{3} \pi r^3 \right) = \left( 2.13 \times 10^{-2} \right) \left( \frac{4}{3} \pi \right) (0.0200 \text{ cm})^3 = 7.13 \times 10^{-7} \text{ C} = 713 \text{nC}
\]

(b) \[
q_m = \rho \left( \frac{4}{3} \pi r^3 \right) = \left( 2.13 \times 10^{-2} \right) \left( \frac{4}{3} \pi \right) (0.0400 \text{ cm})^3 = 5.70 \mu \text{C}
\]

57. A solid, insulating sphere of radius \( a \) has a uniform charge density \( \rho \) and a total charge \( Q \). Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are \( b \) and \( c \), as shown in Figure P24.57. (a) Find the magnitude of the electric field in the regions \( r < a \), \( a < r < b \), \( b < r < c \), and \( r > c \). (b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.

**P24.57**  
(a) \[
\int E \cdot dA = E \left( 4 \pi r^2 \right) = \frac{q_m}{\varepsilon_0}
\]

For \( r < a \),

\[
q_m = \rho \left( \frac{4}{3} \pi r^3 \right)
\]

so

\[
E = \frac{\rho r}{3 \varepsilon_0}.
\]

For \( a < r < b \) and \( c < r \),

\[
q_m = Q.
\]

So

\[
E = \frac{Q}{4 \pi r^2 \varepsilon_0}.
\]

**FIG. P24.57**
For \( b \leq r \leq c \), \( E = 0 \), since \( E = 0 \) inside a conductor.

(b) Let \( q_i \) = induced charge on the inner surface of the hollow sphere. Since \( E = 0 \) inside the conductor, the total charge enclosed by a spherical surface of radius \( b \leq r \leq c \) must be zero.

Therefore, \( q_i + Q = 0 \) and \( \sigma_1 = \frac{q_i}{4\pi b^2} = \frac{-Q}{4\pi b^2} \).

Let \( q_o \) = induced charge on the outside surface of the hollow sphere. Since the hollow sphere is uncharged, we require \( q_i + q_o = 0 \) and \( \sigma_2 = \frac{q_o}{4\pi c^2} = \frac{Q}{4\pi c^2} \).