PROBLEM 11.14

Steam enters the turbine of a power plant at 5 MPa and 400°C and exhausts to the condenser at 10 kPa. The turbine produces a power output of 20,000 kW with an isentropic efficiency of 85%. What is the mass flow rate of steam around the cycle, and the rate of heat rejection in the condenser? Find the thermal efficiency of the power plant. How does this compare with a Carnot cycle?

![Diagram of steam cycle]

\begin{align*}
P_1 &= 10 \text{ kPa} \\
T_3 &= 400°C \\
\dot{W}_T &= 20,000 \text{ kW} \\
P_4 &= 10 \text{ kPa} \\
\dot{W}_p &= \text{ unknown} \\
P_2 &= P_3 = 5 \text{ MPa} \\
P_1 &= P_4 = 10 \text{ kPa} \\
\text{ (At this point, we don't know exactly where state 4a is located.)}
\end{align*}
PROBLEM 11.14 - CONT'D

ASSUMPTIONS

(1) ASSUME RANKINE CYCLE BEHAVIOR
(2) THE TURBINE PERFORMANCE IS NON-IDEAL

<table>
<thead>
<tr>
<th>STATE</th>
<th>$P$ (kPa)</th>
<th>$T$ (°C)</th>
<th>$V$ (m$^3$/kg)</th>
<th>$h$ (kJ/kg)</th>
<th>$s$ (kJ/kgK)</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>10</td>
<td>45.81</td>
<td>0.001010</td>
<td>191.81</td>
<td>0.6472</td>
<td>0.0</td>
</tr>
<tr>
<td>2s</td>
<td>5000</td>
<td>16.1(i)</td>
<td>0.001009(i)</td>
<td>197.8(i)</td>
<td>0.6472</td>
<td>-</td>
</tr>
<tr>
<td>3a</td>
<td>5000</td>
<td>400</td>
<td>0.05781</td>
<td>3195.64</td>
<td>6.6458</td>
<td>-</td>
</tr>
<tr>
<td>4s</td>
<td>10</td>
<td>45.81</td>
<td>11.73</td>
<td>2104.73</td>
<td>6.6458</td>
<td>0.799</td>
</tr>
<tr>
<td>4a</td>
<td>10</td>
<td></td>
<td>2268.36</td>
<td></td>
<td></td>
<td>0.8678</td>
</tr>
</tbody>
</table>

ANALYSIS

FOR THE TURBINE, WE KNOW

\[ W_{T_a} = m (h_3 - h_{4a}) \]
\[ W_{T_s} = m (h_3 - h_{4s}) \]

so

\[ W_{T_a} = (h_3 - h_{4a}) \]

and

\[ W_{T_s} = (h_3 - h_{4s}) \]

WE ALSO KNOW BY DEFINITION

\[ h_T = \frac{W_{T_a}}{W_{T_s}} \]

Thus,

\[ W_{T_a} = h_T W_{T_s} = h_T (h_3 - h_{4s}) \]
PROBLEM 11.14 - CONT'D

\[ W_{Ta} = 0.85 \left( \frac{395.64 \text{kJ}}{\text{kg}} - 2104.73 \frac{\text{kJ}}{\text{kg}} \right) \]

\[ W_{Ta} = 927.28 \text{ kJ/kg} \quad \rightarrow \]

so

\[ h_{4a} = h_3 - W_{Ta} = \frac{395.64 \text{kJ}}{\text{kg}} - 928.24 \frac{\text{kJ}}{\text{kg}} \]

\[ h_{4a} = 2268.36 \text{ kJ/kg} \quad \rightarrow \]

from Table 2.1.2.0 10 kPa

\[ h_f|_{10\text{kPa}} < h_{4a} < h_g|_{10\text{kPa}} \]

so state 4a is in the two-phase region, and

\[ \chi_{4a} = \frac{h_{4a} - h_f|_{10\text{kPa}}}{h_g|_{10\text{kPa}} - h_f|_{10\text{kPa}}} = \frac{2268.36 - 191.81}{2392.82} \]

\[ \chi_{4a} = 0.8678 \quad \rightarrow \]

now, since

\[ \dot{W}_{Ta} = m \dot{W}_{Ta} \]

\[ m = \frac{\dot{W}_{Ta}}{W_{Ta}} = \frac{20,000 \text{ kW}}{927.28 \text{ kJ/kg}} \]

\[ m = 21.57 \text{ kg/s} \quad \rightarrow \]

for the condenser

\[ \dot{Q}_c = \dot{m} (h_1 - h_4) \]

\[ = 21.57 \text{ kg/s} \left( 191.81 \frac{\text{kJ}}{\text{kg}} - 2268.36 \frac{\text{kJ}}{\text{kg}} \right) \]
PROBLEM 11.14 - CONT'D

\[ Q_c = -44,791.2 \text{ kW} \]

By definition, the thermal efficiency is

\[ \eta_{TH} = \frac{\text{NETWORK}}{\text{ENERGY REQUIRED}} = \frac{-\dot{W}_{Ta} + \dot{W}_p}{\dot{Q}_{BOILER}} = \eta^c \left( \frac{\dot{W}_{Ta} + \dot{W}_p}{\dot{Q}^c_{BOILER}} \right) \]

\[ \eta_{TH} = \frac{\dot{W}_{Ta} + (h_1 - h_2s)}{(h_3 - h_2s)} \]

\[ = \frac{927.28 \text{ kJ/kg} + \left( 191.81 \text{ kJ/kg} - 177.38 \text{ kJ/kg} \right)}{(3195.64 \text{ kJ/kg} - 197.38 \text{ kJ/kg})} \]

\[ \eta_{TH} = 0.3074 \]

The Carnot cycle efficiency is defined as

\[ \eta_{TH,c} = 1 - \frac{T_L}{T_H} = 1 - \frac{T_{BOILER}}{T_{HOT}} \]

\[ = 1 - \frac{318.96 K}{673.15 K} \]

\[ \eta_{TH,c} = 0.526 \]

Note: We could have found the pump work using

\[ \dot{W} = \nu (P_1 - P_2) \]
PROBLEM 11.14 - CONTO

Which is valid for a reversible, adiabatic device operating using an incompressible fluid. In the case,

\[ \dot{W}_p = 0.001010 \frac{m^3}{kg} (4990 \text{ kPa}) \]

\[ \dot{W}_p = -5.04 \frac{kJ}{kg} \leftarrow \]

Using tabulated enthalpies, we found

\[ \dot{W}_p = -5.57 \frac{kJ}{kg} \leftarrow \]

The difference is a result of error introduced by the assumption of incompressibility.
PROBLEM 11.21

In one type of nuclear power plant, heat is transferred in the nuclear reactor to liquid sodium. The liquid sodium is then pumped through a heat exchanger where heat is transferred to boiling water. Saturated vapor steam at 5 MPa exits this exchanger and is then superheated to 600°C in an external gas-fired superheater. The steam enters the turbine, which has one (open type) feedwater extraction at 0.4 MPa. The condenser pressure is 7.5 kPa. Determine the heat transfer in the reactor and in the superheater to produce a net power output of 1 MW.

\[ Q_{\text{superheater}} \]

\[ \dot{q}_{\text{boiler}} \]

\[ \dot{W}_{\text{net}} = 1 \text{ MW} \]

Assumptions

1. The system and its components operate as an ideal regenerative cycle.
**Problem 11.21 - Cont'd**

### Table

<table>
<thead>
<tr>
<th>STATE</th>
<th>( \overline{V} ) (m³/kg)</th>
<th>( P ) (kPa)</th>
<th>( T ) (°C)</th>
<th>( h ) (kJ/kg)</th>
<th>( s ) (kJ/kgK)</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.001008</td>
<td>7.5</td>
<td>40.29</td>
<td>168.77</td>
<td>0.5763</td>
<td>0.0</td>
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<td>2</td>
<td>0.001008</td>
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<td></td>
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</tr>
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<td>144.17(i)</td>
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<tr>
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</tr>
<tr>
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<td>0.56002 (i)</td>
<td>400</td>
<td>221.2(i)</td>
<td>2904.42(i)</td>
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<td></td>
</tr>
<tr>
<td>8</td>
<td>16.75</td>
<td>7.5</td>
<td>40.29</td>
<td>2263.69</td>
<td>7.2588</td>
<td>0.8707</td>
</tr>
</tbody>
</table>

- **Double-underlined values are given**
- **Single-underlined values are found from tables directly from double-underlined values**

**Analysis**

For a control volume encompassing the pump and assuming:

1. **Reversible**
2. **Adiabatic**
PROBLEM 11.21 - CONT'D

(3) NEGLIGIBLE ΔKE AND ΔPE
(4) STEADY FLOW
(5) INCOMPRESSIBLE FLUID \( (\nu_1 = \nu_2) \)

THE FIRST AND SECOND LAWS YIELD

\[ \cdot W_p = m_p (h_3 - h_4) \]  \hspace{1cm} (1)

\[ W_p = (h_3 - h_4) = -5.34 \text{ kJ/kg} \] \hspace{1cm} (2)

AND

\[ W_p = \left. \left( \int_{v_3}^{v_4} \nu dp = \int_{v_3}^{v_4} d\rho = \nu \Delta \rho \right) \right|_{v_3}^{v_4} \]  \hspace{1cm} (3)

\[ W_p = -0.001008 \frac{\text{m}^3}{\text{kg}} (400 \text{kPa} - 500 \text{kPa}) \]

\[ W_p = -4.637 \text{ kJ/kg} \] \hspace{1cm} (4)

(THE DIFFERENCE BETWEEN THE CALCULATED VALUES IS DUE TO THE ASSUMPTION OF INCOMPRESSIBLE BEHAVIOR IN EQUATION (3). THE VALUE OBTAINED USING TABULATED VALUES, EQUATION (2) IS MOST ACCURATE.)

TO DEFINE STATE \( (8s) \) WE NOTE

\[ S_f |_{7.5 \text{kPa}} < S_{(8s)} < S_g |_{7.5 \text{kPa}} \]

THUS

\[ x_{\text{(8s)}} = \frac{S_{8s} - S_f |_{8s}}{S_{fg} |_{8s}} \]  \hspace{1cm} (5)

\[ = \frac{7.2588 - 0.5763}{7.6751} \]

\[ x_{\text{(8s)}} = 0.8707 \]
**PROBLEM 11.21 - CONT'D**

For control volumes around the boiler and superheater, assuming

1. **steady flow**
2. **negligible ΔKE and ΔPE**
3. **no shaft work**
4. **one inlet/one exit**

We find

\[
\dot{Q}_{\text{boiler}} = \dot{m}_{\text{steam}} (h_5 - h_{4S}) \tag{6}
\]

\[
\dot{Q}_{\text{superheat}} = \dot{m}_{\text{steam}} (h_6 - h_5) \tag{7}
\]

To evaluate Eqs (6) and (7) we must find the flow rate of steam.

For a control volume encompassing the turbine, assuming

1. **steady flow**
2. **negligible ΔKE and ΔPE**
3. **adiabatic**
4. **one inlet/two exits**

We find, applying the first law:

\[
\dot{W}_T = \dot{m}_{\text{steam}} h_6 - \dot{m}_{\text{extract}} h_{7S} - \dot{m}_{\text{condense}} h_{8S} \tag{8}
\]

\[
\dot{W}_T = h_6 - \left( \frac{\dot{m}_{\text{extract}}}{\dot{m}_{\text{steam}}} \right) h_{7S} - \left( \frac{\dot{m}_{\text{condense}}}{\dot{m}_{\text{steam}}} \right) h_{8S} \tag{9}
\]

Applying continuity:

\[
\dot{m}_{\text{steam}} = \dot{m}_{\text{extract}} + \dot{m}_{\text{condense}} \tag{10}
\]

\[
1 = \left( \frac{\dot{m}_{\text{extract}}}{\dot{m}_{\text{steam}}} \right) + \left( \frac{\dot{m}_{\text{condense}}}{\dot{m}_{\text{steam}}} \right) \tag{11}
\]
DEFINING A CONTROL VOLUME AROUND THE F EEDW A TER HE A TER, AND ASSUMING

(1) STEADY FLOW
(2) NEGLIGIBLE ΔKE AND ΔP
(3) NO SHAFT WORK
(4) ADIABATIC
(5) TWO INLETS/ONE EXIT

APPLYING THE FIRST LAW:

\[ \dot{m}_{\text{water}} h_3 = \dot{m}_{\text{extract}} h_{7S} + \dot{m}_{\text{condense}} h_{2S} \quad (12) \]

\[ h_3 = \left( \frac{\dot{m}_{\text{extract}}}{\dot{m}_{\text{water}}} \right) h_{7S} + \left( \frac{\dot{m}_{\text{condense}}}{\dot{m}_{\text{water}}} \right) h_{2S} \quad (13) \]

NOTING \( \dot{m}_{\text{water}} \) IS EQUAL TO \( \dot{m}_{\text{steam}} \), EQUATION (13) AND EQUATION (11) CAN BE COMBINED TO GIVE

\[ h_3 = \left( 1 - \frac{\dot{m}_{\text{condense}}}{\dot{m}_{\text{steam}}} \right) h_{7S} + \left( \frac{\dot{m}_{\text{condense}}}{\dot{m}_{\text{steam}}} \right) h_{2S} \]

\[ \frac{\dot{m}_{\text{condense}}}{\dot{m}_{\text{steam}}} (h_{2S} - h_{7S}) = h_3 - h_{7S} \quad (14) \]

\[ \frac{\dot{m}_{\text{condense}}}{\dot{m}_{\text{steam}}} = \frac{h_3 - h_{7S}}{h_{2S} - h_{7S}} = 0.8408 \quad (15) \]

THUS, FROM EQUATION (11)

\[ \frac{\dot{m}_{\text{extract}}}{\dot{m}_{\text{steam}}} = 0.15924 \]

WE CAN NOW FIND THE WORK DONE BY THE
PROBLEM 11.21 - CONT'D

TURBINE FROM EQN (4) AS

\[ W_T = 3666.47 \frac{kJ}{kg} - 0.15924 (2904.42 \frac{kJ}{kg}) \]
\[ - 0.8408 (226.319 \frac{kJ}{kg}) \]
\[ W_T = 1301.08 \frac{kJ}{kg} \]

FINALLY, FOR THE FEEDWATER PUMP

\[ \dot{W}_{FWP} = m_{CONDENSED} (h_1 - h_{2s}) \] (16)
\[ \dot{W}_{FWP} = (h_1 - h_{2s}) \] (17)

TO FIND STATE (2) WE ASSUME INCOMPRESSIBLE FLUID BEHAVIOR, THUS

\[ W_{FWP} = \dot{V} \Delta P = \dot{V} (P_1 - P_{2s}) \]
\[ = 0.001008 \frac{m^3}{kg} (7.5 \text{kPa} - 400 \text{kPa}) \]
\[ = -0.3956 \frac{kJ}{kg} \] (18)

THEN USING EQN (17):

\[ -0.3956 \frac{kJ}{kg} = h_1 - h_{2s} \]
\[ h_{2s} = 169.17 \frac{kJ}{kg} \]

NOW, THE WORK DONE BY THE SYSTEM IS

\[ \dot{W}_{NET} = \dot{W}_T + \dot{W}_{FWP} + \dot{W}_P \]
\[ = m_{STEAM} \dot{W}_T + m_{CONDENSED} \dot{W}_{FWP} + m_{STEAM} \dot{W}_P \]
PROBLEM 11.21 - CONT'D

SO

\[ \dot{W}_{\text{NET}} = \dot{W}_T + \dot{m}_{\text{condense}} \dot{W}_{\text{fwp}} + \dot{W}_P \]

\[ = \frac{1301.08 \text{kJ}}{\text{kg}} + 0.8408(-0.3956 \text{kJ/kg}) - 5.34 \text{kJ/kg} \]

\[ \frac{\dot{W}_{\text{NET}}}{\dot{m}_{\text{steam}}} = 1295.41 \text{kJ/kg} \]

SO

\[ \dot{m}_{\text{steam}} = 0.772 \text{kg/s} \]

AND FROM EQNS (6) AND (7)

\[ \dot{Q}_{\text{boiler}} = 1686.2 \text{kw} \]

\[ \dot{Q}_{\text{superheater}} = 673.3 \text{kw} \]

THE THERMAL EFFICIENCY IS

\[ \eta_{\text{th}} = \frac{\dot{W}_{\text{NET}}}{\dot{Q}_H} = \frac{1000 \text{kw}}{1686.2 \text{kw} + 673.3 \text{kw}} \]

\[ \eta_{\text{th}} = 0.424 \]