A compressor brings R-410a (see Problem 10.22) from -10°C and 125 kPa up to 500 kPa in an adiabatic reversible compression. Assume ideal gas behavior and find the exit temperature and the specific work.

**Assumptions**

1. Steady flow
2. Reversible
3. Adiabatic
4. Negligible ΔKE, ΔPE
5. R410A composed of R32/R-125 in a 1:1 ratio on a mass basis (from Problem 10.22)
6. Since no tables for R-32 or R-125 are given, assume constant specific heats.

**Solution**

The first law applied to the compressor is:

$$\dot{Q}_{cv} + \sum \dot{m}_i (h_i + \frac{\dot{V}_i^2}{2} + \frac{\dot{E}_i^2}{2}) = \sum \dot{m}_e (h_e + \frac{\dot{V}_e^2}{2} + \frac{\dot{E}_e^2}{2}) + \dot{W}_{cv}$$

Since there is only one inlet and one exit,

$$\dot{m}_i = \dot{m}_e = \dot{m}$$

and the first law yields:

$$-\dot{W}_{cv} = \dot{m} (h_e - h_i)$$

$$-\dot{W}_{cv} = (h_e - h_i)$$

(1)

Can we assume ideal gas behavior?
**PROBLEM 10.29 - CONT'D**

From Table A.2:

\[ T_{c-32} |_{R-32} = 351.3 \text{ K} \quad P_{c-32} |_{R-32} = 5.78 \text{ MPa} \]

\[ T_{c-125} |_{R-125} = 339.12 \text{ K} \quad P_{c-125} |_{R-125} = 3.62 \text{ MPa} \]

We know the total pressure of R-410A initially is 125 kPa. Therefore,

\[ R |_{R-410A} = C_{R-32} R_{R-32} + C_{R-125} R_{R-125} = 0.114548 \frac{\text{kJ}}{\text{kg K}} \]

\[ M_w |_{R-410A} = 72.59 \text{ kg mol}^{-1} \]

Then,

\[ Y_i = \frac{C_i (M_w)}{(M_w)} |_{R-410A} \]

so

\[ Y_{R-32} = 0.698 \quad Y_{R-125} = 0.302 \]

And

\[ T_{R-32} = 0.75 \quad P_{R-32} = 0.0065 \]

\[ T_{R-125} = 0.78 \quad P_{R-125} = 0.0104 \]

So the gas is **ideal**. Therefore,

\[ h = f(T) = \int C_p \, dT \]

Assuming constant specific heats

\[ \Delta h = C_p \Delta T \quad (2) \]

From the first and second laws

\[ -\omega_{cu} = C_p \left( T_c - T_i \right) \quad (3) \]

\[ \Delta S = S^0_{T_2} - S^0_{T_1} - R \ln \left( \frac{P_2}{P_1} \right) \]

But for constant specific heats

\[ \Delta S = 0 = C_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right) \quad (4) \]
Problem 10.29 - Cont'd

\[ \Delta S = 0 = C_{p0} \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right) \]

\[ \ln \left( \frac{T_2}{T_1} \right) = \frac{R}{C_{p0}} \ln \left( \frac{P_2}{P_1} \right) \]

\[ T_2 = \frac{\left( \frac{P_2}{P_1} \right)^{R/C_{p0}}}{T_1} \]

\[ T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{R/C_{p0}} \] \hspace{1cm} (5)

Now, if we assume the Dalton Model of gas mixtures is valid:

\[ C_{p0} \mid_{\text{mix}} = C_{R-32} C_{p0 R-32} + C_{R-125} C_{p0 R-125} \]

AND

\[ R \mid_{\text{mix}} = C_{R-32} R_{R-32} + C_{R-125} R_{R-125} \]

Yielding:

\[ C_{p0} \mid_{\text{mix}} = \left( \frac{1 \text{ kg}}{2 \text{ kg}} \right) \left( 0.822 \text{ K} \cdot \text{kJ}^{-1} \right) + \left( \frac{1 \text{ kg}}{2 \text{ kg}} \right) \left( 0.791 \text{ K} \cdot \text{kJ}^{-1} \right) = 0.8065 \text{ K} \cdot \text{kJ}^{-1} \]

\[ R_{\text{mix}} = \left( \frac{1 \text{ kg}}{2 \text{ kg}} \right) \left( \frac{8.3145 \text{ K} \cdot \text{kJ}}{52.074 \text{ kg} \cdot \text{kJ} \cdot \text{mol}^{-1}} \right) + \left( \frac{1 \text{ kg}}{2 \text{ kg}} \right) \left( \frac{8.3145 \text{ K} \cdot \text{kJ}}{120.022 \text{ kg} \cdot \text{kJ} \cdot \text{mol}^{-1}} \right) \]

\[ R_{\text{mix}} = 0.11455 \text{ K} \cdot \text{kJ}^{-1} \]

\[ C_{p0} \mid_{\text{mix}} = 0.8065 \text{ K} \cdot \text{kJ}^{-1} \]
Problem 10.29 - cont'd

Solving Eqs. (5) and (3) we find

\[ T_2 = 320.4 \text{K} \]

and

\[ \omega = -46.1 \frac{\text{kJ}}{\text{kg}} \]
Problem 10.42

Atmospheric air is at 100 kPa and 25°C with a relative humidity of 75%. Find the absolute humidity and the dew point of the mixture. If the mixture is heated to 30°C what is the new relative humidity?

Assumptions

(1) The Dalton model of mixtures is valid.
(2) The partial pressure of water vapor (over the liquid) is that of the saturation pressure corresponding to the mixture temperature.

Solution

The specific humidity (or "humidity ratio" or "absolute humidity") is defined as:

\[ \omega = \frac{m_w}{m_a} = 0.622 \frac{P_w}{P_a} \]  

(1)

And the relative humidity is

\[ \phi = \frac{P_w}{P_g} \]  

(2)

So

\[ \omega = 0.622 \frac{\phi P_g}{P_a} \]  

(3)

From the problem statement, we know

\[ \phi = 0.75 \]  

and, from Table B.1.1,

\[ P_g = 3.169 \text{ kPa} \]

therefore, the vapor pressure is

\[ P_w = \phi P_g = 2.377 \text{ kPa} \]

and the dry air pressure is

\[ P_a = P_{\text{total}} - P_w = 97.623 \text{ kPa} \]
**Problem 10.42 - Cont'd**

Then the humidity ratio is, from Eqn (3),

\[
\omega = 0.622 \frac{2.377 \text{kPa}}{97.623 \text{kPa}}
\]

\[
\omega = 0.01514 \frac{\text{kg} \omega}{\text{kg} \text{a}}
\]

The dew point is the saturation temperature corresponding to the partial pressure of the water vapor. From Table B.1.2.

\[
\theta_{dp} = 20.2^\circ \text{C}
\]

2.377 kPa

If the mixture is heated, and no water is added or removed,

\[
\omega \bigg|_{25^\circ \text{C}} = \omega \bigg|_{30^\circ \text{C}}
\]

And since the air pressure is constant

\[
P_a \bigg|_{25^\circ \text{C}} = P_a \bigg|_{30^\circ \text{C}}
\]

But now, using Table B.1.1

\[
P_g = 4.246 \text{ kPa}
\]

So

\[
\phi \bigg|_{30^\circ \text{C}} = \frac{2.377 \text{kPa}}{4.246 \text{kPa}}
\]

\[
\phi \bigg|_{30^\circ \text{C}} = 0.560
\]

So the heating results in a decreased relative humidity while specific humidity remains constant.
PROBLEM 10.70

Ambient air at 100 kPa, 30°C, and 40% relative humidity goes through a constant-pressure heat exchanger as a steady flow. In one case it is heated to 45°C and in another case it is cooled until it reaches saturation. For both cases find the exit relative humidity and the amount of heat transfer per kilogram of dry air.

ASSUMPTIONS

1. Steady-flow
2. Negligible ΔKE, ΔPE
3. No shaft work
4. Ideal gas behavior
5. Dalton model of mixtures is valid

SOLUTION

The first law applied to the control volume yields:

\[ Q_{cu} + \sum m_i (h_i + \frac{V_i^2}{2} + g z_i) = \sum \dot{m}_e (h_e + \frac{V_e^2}{2}) + \dot{W}_{cu} \]

Since there is only one inlet and one exit

\[ Q_{cu} = \dot{m} (h_1 - h_2) \]

\[ q = (h_1 - h_2) \quad (1) \]

Can we find the enthalpies and relative
PROBLEM 10.70-CONT'D

HUMIDITIES USING THE PSYCHROMETRIC CHART?

CASE (1)

\[ T_1 = 30^\circ C \]
\[ \varphi_1 = 40\% \]
\[ P_1 = P_2 = 100 \text{ kPa} \]
\[ T_2 = 45^\circ C \]

FROM FIGURE E.4

\[ h_1 = 76.0 \text{ kJ/kg} \]

(From ASHRAE CHART)

(From ASHRAE CHART)

\[ h_1 = 57.2 \text{ kJ/kg} \]

Since water is not added or removed from the flow:

\[ w_1 = w_2 \]

FROM FIGURE E.4

\[ w = 0.011 \text{ kg} \text{J/kg} \]

FROM FIGURE E.4

\[ \varphi_2 = 17\% \]

\[ h_2 = 91.4 \text{ kJ/kg} \]

(From ASHRAE CHART)

(From ASHRAE CHART)

\[ h_1 = 72.5 \text{ kJ/kg} \]

CASE (2)

\[ T_1 = 30^\circ C \]
\[ \varphi_1 = 40\% \]
\[ P_1 = P_2 = 100 \text{ kPa} \]

FROM FIGURE E.4

\[ h_1 = 77.0 \text{ kJ/kg} \]

(From ASHRAE CHART)

\[ h_1 = 57.2 \text{ kJ/kg} \]

Since state 2 is saturated

\[ T_2 = T_{saturation} \]

Therefore,

\[ \varphi_2 = 100\% \]

Since water is not added or removed from the flow

\[ w_1 = w_2 \]

FROM FIGURE E.4

\[ h_2 = 60.5 \text{ kJ/kg} \]

\[ T_2 = 14.5^\circ C \]

(From ASHRAE CHART)

(From ASHRAE CHART)

\[ h_2 = 42.1 \text{ kJ/kg} \]
**PROBLEM 10.70 - CONT'D**

**CASE (1)**

From Eqn. (1)

\[ q_t = (h_2 - h_1) \]

\[ q_t = \frac{91.4 \text{ KJ}}{\text{kja}} - \frac{76.0 \text{ KJ}}{\text{kja}} \]

\[ q_t = 15.4 \text{ KJ/kja} \]

Using the values from the ASHRAE chart

\[ q_t = 15.3 \text{ KJ/kja} \]

**CASE (2)**

From Eqn (1)

\[ q_t = (h_2 - h_1) \]

\[ q_t = \frac{60.5 \text{ KJ}}{\text{kja}} - \frac{77.0 \text{ KJ}}{\text{kja}} \]

\[ q_t = -16.5 \text{ KJ/kja} \]

Using the values from the ASHRAE chart:

\[ q_t = -15.1 \text{ KJ/kja} \]

Notice that the reference basis for the enthalpies in the texts chart compared to ASHRAE's chart are different, but the enthalpy differences are the same. (The slight differences between the ASHRAE and text-derived answer for the two cases can be attributed to uncertainties associated with reading the charts.)