PROBLEM 6.80

A steam engine based on a turbine is shown below. The boiler tank has a volume of 100 L and initially contains saturated liquid with a very small amount of vapor at 100 kPa. Heat is now added by the burner. The pressure regulator, which keeps pressure constant, does not open before the boiler pressure reaches 700 kPa, and is discharged to the atmosphere as saturated vapor at 100 kPa. The burner is turned off when no more liquid is present in the boiler. Find the total turbine work and the total heat transfer to the boiler for this process.

ASSUMPTIONS

(1) TRANSIENT PROCESS
(2) REGULATOR DOESN'T OPEN UNTIL \( P_{H_2O} = 700 \text{ kPa} \)
(3) NEGLIGIBLE ΔKE AND ΔPE IN THE PROCESS

SOLUTION

To begin, apply the continuity and energy equations to a control volume encompassing the boiler (C.V.),

\[
\frac{dm_{cv}}{dt} = \Sigma m_i - \Sigma m_e
\]

For this C.V., there is only one exit, so

\[
\frac{dm_{cv}}{dt} = -m_e \tag{1}
\]

The energy eqn yields

\[
\frac{dE_{cv}}{dt} = Q_{cv} - W_{cv} + \sum m_i(h_i + \frac{V_i^2}{2} + gz_i) + \sum m_e(h_e + \frac{V_e^2}{2} + gz_e)
\]
PROBLEM 6.80 - CONT'D

NOTING THERE IS NO SHAFT WORK DONE ON THE C. F. AND EXPANDING THE TOTAL ENERGY TERM,

\[
\frac{d}{dt} \left( U_{cu} + \frac{mv^2}{2} + mgz \right) = Q_{cu} - m e \cdot h e \quad (2)
\]

INTEGRATING

\[
dU_{cu} = Q_{cu} \, dt - m e h e \, dt
\]

BUT FROM CONTINUITY (EQU. 1) WE KNOW

\[
m_{cu} \left|_1 \right. = m_{cu} \left|_2 \right. - m_{cu} \left|_1 \right. = -m_c
\]

SO

\[
U_{cu} \left|_1 \right. = Q_{cu} - m e h e
\]

\[
U_{cu} \left|_2 \right. - U_{cu} \left|_1 \right. = Q_{cu} + \left( m_{cu} \left|_2 \right. - m_{cu} \left|_1 \right. \right) h e
\]

SOLVING FOR THE HEAT TRANSFER TO THE BUILDER,

\[
Q_{cu} = m_{cu} \left|_2 \right. (U_2 - h e) - m_{cu} \left|_1 \right. (U_1 - h e) \quad (3)
\]

WE KNOW STATES 1 AND 2. FROM TABLE B.1.2.

\[
U_f \left|_{100 \text{kPa}} \right. = U_1 = 0.001043 \text{ m}^3/\text{kg}
\]

SO

\[
m_{cu} \left|_1 \right. = \frac{V_1}{U_1} = \frac{0.1 \text{ m}^3}{0.001043 \text{ m}^3/\text{kg}} = 95.88 \text{ kg} \quad \leftarrow
\]

AND

\[
U_f \left|_{100 \text{kPa}} \right. = U_1 = 417.33 \text{ kJ/kg} \quad \leftarrow
\]

FOR STATE 2, WE HAVE SATURATED VAPOR (x=1.0) AT 700 kPa. FROM TABLE B.1.2

\[
U_g \left|_{700 \text{kPa}} \right. = U_2 = 0.27786 \text{ m}^3/\text{kg} \quad \leftarrow
\]
\[ m_{cu} \mid _2 = \frac{\frac{V}{U_2}}{0.22786 \text{ m}^3} = 0.3665 \text{ kg} \]

and

\[ U_g \mid _{700 \text{ kPa}} = U_2 = 2572.49 \text{ kJ/kg} \]

Substituting this information into Eqn. (3) and noting that only vapor leaves the turbine,

\[ Q_{cu} = 0.3665 \text{ kg} \left( \frac{2572.49 \text{ kJ}}{\text{kg}} - 2763.50 \frac{\text{kJ}}{\text{kg}} \right) \]

\[ - 95.88 \text{ kJ} \left( \frac{417.33 \text{ kJ}}{\text{kg}} - 2763.50 \frac{\text{kJ}}{\text{kg}} \right) \]

\[ Q_{cu} = Q_{boiler} = 224,881 \text{ kJ} \]

Now we must find the total turbine work. Drawing a control volume around the boiler, regulator, and turbine (Unit 2) and applying the continuity and energy Eqs., yields

\[ \frac{dm_{cu}}{dt} = -m_e \]  \hspace{1cm} (4)

\[ \frac{dU_{cu}}{dt} = \dot{Q}_{cu} - \dot{W}_{cu} - m_e \dot{h}_{e} \]  \hspace{1cm} (5)

Integrating,

\[ dU_{cu} = \dot{Q}_{cu} dt - \dot{W}_{cu} dt - m_e \dot{h}_{e} dt \]  \hspace{1cm} (6)

Since

\[ m_{cu} \mid _2 - m_{cu} \mid _1 = -m_e \]

we find we integrating Eqn. (6)
PROBLEM 6.80 - CONT'D

\[ u_{C_2} - u_{C_1} = Q_{cv} - W_{cv} + (mc_{v_2} - mc_{v_1})he \quad (7) \]

Now, if we assume the turbine and regulator are insulated

\[ Q_{cv} = Q_{boiler} \]

Noting that \( W_{cv} \) is the shaft work done by the turbine

\[ W_{turb} = Q_{boiler} + (mc_{v_2} - mc_{v_1})he \]
\[ + mc_{v_1}u_{C_1} - mc_{v_2}u_{C_2} \]

\[ W_{turb} = Q_{boiler} + mc_{v_2}(he - u_{C_2}) \]
\[ + mc_{v_1}(u_{C_1} - he) \quad (8) \]

Evaluating Eqn (8)

\[ W_{turb} = 224,881 \text{ KJ} + 0.3665 \text{ KJ} \left( \frac{2675.46 \text{ KJ}}{\text{KJ}} - 2572.49 \text{ KJ} \right) \]
\[ - 95.88 \text{ KJ} \left( \frac{2675.46 \text{ KJ}}{\text{KJ}} - 417.33 \text{ KJ} \right) \]

\[ W_{turb} = 8409 \text{ KJ} \]

(A choice of a C.V. surrounding only the turbine would not have been helpful. Can you show why?)
PROBLEM 6.51

A superheater brings 2.5 kg/s saturated water vapor at 2 MPa to 450°C. The energy is provided by hot air at 1200 K flowing outside the steam tube in the opposite direction as the water, a setup known as a counterflowing heat exchanger. Find the smallest possible mass flow rate of the air to ensure that its exit temperature is 20°C larger than the incoming water temperature.

ASSUMPTIONS

(1) STEADY STATE

(2) NEGUGIBLE ΔKE AND ΔPE

(3) NO SHAFT WORK

(4) ADIABATIC PROCESS FOR C.F. SHOWN (THIS WOULD NOT BE THE CASE FOR C.F.'S AROUND THE SEPARATE STREAMS.)

(5) CONSTANT PRESSURE WATER FLOW, P₂ = P₁

(6) AIR BEHAVES AS AN IDEAL GAS. (THIS IS PROBABLY TRUE FOR A HEAT EXCHANGER SINCE T₂ ≫ 1 AND PR IS LIKELY ≫ 1.)

SOLUTION

APPLYING THE CONTINUITY EQUATION TO BOTH STREAMS SEPARATELY:

\[ m_3 = m_4 = m_{air} \]
\[ m_1 = m_2 = m_{H₂O} \]

APPLYING THE ENERGY EQUATION TO THE CONTROL VOLUME IDENTIFIED YIELDS:
PROBLEM 6.51 - CONT'D

\[
\frac{dE_{sv}}{dt} = o_{(1)} z_{2} - \sum m_{i}(h_{i} + \frac{V_{i}^{2}}{2} + gZ_{i}) - \sum m_{e}(he + \frac{V_{e}^{2}}{2} + gze)
\]

EXPANDING

\[
m_{\text{Air}} h_{4} + m_{\text{H}_2\text{O}} h_{2} = m_{\text{Air}} h_{3} + m_{\text{H}_2\text{O}} h_{1}
\]

COLLECTING TERMS

\[
m_{\text{Air}} (h_{4} - h_{3}) = m_{\text{H}_2\text{O}} (h_{1} - h_{2})
\]

\[
\frac{m_{\text{Air}}}{m_{\text{H}_2\text{O}}} = \frac{(h_{1} - h_{2})}{(h_{4} - h_{3})} \tag{1}
\]

NOW, FROM TABLE A.7.1

\[
h_{3} = 1277.81 \text{ kJ/kg}
\]

AND FROM TABLE B.1.1

\[
T_{1} = 212.42^\circ C \quad h_{1} = 2799.51 \text{ kJ/kg}
\]

SO

\[
T_{4} = 232.42^\circ C = 505.57^\circ K
\]

SINCE WE ASSUMED IDEAL BEHAVIOR FOR THE AIR, FROM TABLE A.7.1, BY INTERPOLATION

\[
h_{4} = 509.1 \text{ kJ/kg}
\]

PLUGGING THESE VALUES INTO EQUATION (1) YIELDS

\[
m_{\text{Air}} = 2.5 \text{ kg/s} \left( \frac{2799.51 \text{ kJ/kg} - 3357.48 \text{ kJ/kg}}{509.1 \text{ kJ/kg} - 1277.81 \text{ kJ/kg}} \right)
\]

\[
m_{\text{Air}} = 1.815 \text{ kg/s}
\]
PROBLEM 6.75

A 100 L STAINLESS STEEL TANK CONTAINS CARBON DIOXIDE GAS AT 1 MPa AND 300 K. A VALVE IS CRACKED OPEN, AND CARBON DIOXIDE ESCAPES SLOWLY UNTIL THE TANK PRESSURE HAS DROPPED TO 500 kPa. AT THIS POINT THE VALVE IS CLOSED. THE GAS REMAINING IN THE TANK MAY BE ASSUMED TO HAVE UNDERGONE A POLYTROPIC EXPANSION, WITH POLYTROPIC EXPONENT \( n = 1.15 \). FIND THE FINAL MASS INSIDE AND THE HEAT TRANSFERRED TO THE TANK DURING THE PROCESS.

ASSUMPTIONS

(1) ONE EXIT, NO INLETS
(2) NEGLECTIBLE \( \Delta KE \) AND \( \Delta PE \)
(3) NO WORK DONE
(4) IS THE CO\(_2\) IDEAL?

\[ \frac{p_1}{T_1} = \frac{1}{7.38} = 0.135 \]

\[ T_{c1} = 300/304.1 = 1.0 \]

SO FROM FIGURE 10.1, STATE 1 CAN BE CONSIDERED IDEAL. WE MAY ALSO ASSUME IDEAL BEHAVIOR AT STATE 2 AND CHECK THIS ASSUMPTION LATER (WHEN WE FIND \( T_2 \)).

(5) POLYTROPIC PROCESS, \( p^\gamma = C \)
(6) RIGID TANK, \( \dot{V}_1 = \dot{V}_2 \)

SOLUTION

THE ENERGY EQUATION FOR THE PROCESS IS

\[ \sum_{i} \left( \bar{h}_i + \frac{\nu_i^2}{2} + \frac{\xi_i}{\gamma} \right) - \bar{h}_{ce} = \]

\[ \sum_{m0} \left( \bar{h}_0 + \frac{\nu_0^2}{2} + \frac{\xi_0}{\gamma} \right) - \bar{h}_{ce} = \]

\[ \dot{m}_2 \left( \bar{h}_2 + \frac{\nu_2^2}{2} + \frac{\xi_2}{\gamma} \right) + \dot{m}_2 \left( \bar{h}_2 + \frac{\nu_2^2}{2} + \frac{\xi_2}{\gamma} \right) - \dot{m}_1 \left( \bar{h}_1 + \frac{\nu_1^2}{2} + \frac{\xi_1}{\gamma} \right) \]
\[ Q_{cv} = m_e h_e + m_2 u_2 - m_1 u_1 \]  \hspace{1cm} (1)

**From the Continuity Eqn:**

\[ m_2 - m_1 = -m_e \]  \hspace{1cm} (2)

**Using the Ideal Gas Law and the Polytropic Relation**

\[ P_2^n = c \]
\[ P_1 V_1^n = P_2 V_2^n \]
\[ \frac{V_2}{V_1} = \left( \frac{P_1}{P_2} \right)^{\frac{1}{n}} \]
\[ \frac{T_2}{P_2} \left( \frac{P_1}{T_1} \right)^{\frac{n}{n-1}} = \left( \frac{P_1}{P_2} \right)^{\frac{n}{n-1}} \]
\[ T_2 = \frac{300 K \left( \frac{1}{2} \right)^{0.15/1.15}}{274.1 K +} \]

*(And we now know state 2 is also ideal.)*

**So**

\[ m_1 = \frac{P_1 V_1}{RT_1} = \frac{(1000 \text{ kPa})(100 \text{ L})(0.1 \text{ m}^3)}{(8.3145 \text{ kJ/kmol K})(300 \text{ K})} \cdot \frac{(44.0 \text{ kJ/kmol})}{(44.0 \text{ kJ/kmol})} \]

\[ m_1 = 1.7640 \text{ kg} \]

**And**

\[ m_2 = \frac{(500 \text{ kPa})(0.1 \text{ m}^3)(44.0 \text{ kJ/kmol})}{(8.3145 \text{ kJ/kmol K})(274.1 K)} \]

\[ m_2 = 0.965 \text{ kg} \]
Problem 6.75 - Cont’d

From Eqn. (2)
\[
m_2 = 0.799 \text{ kg}
\]

Now we use Table A.8 for the properties of Ideal Carbon Dioxide:
\[
U_1 = 157.70 \text{ kJ/kg}, \quad h_1 = 214.38 \text{ kJ/kg}
\]
\[
U_2 = 141.39 \text{ kJ/kg}, \quad h_2 = 193.17 \text{ kJ/kg}
\]

What can we do to estimate \( h_e \)?
Since \( h_e = f(T) \) we might assume
\[
\frac{h_1 + h_2}{2} = 203.78 \text{ kJ/kg}
\]

Plugging these values into the energy Eqn gives:
\[
Q_{cu} = 0.799 \left( \frac{203.78 \text{ kJ/kg}}{2} \right) + 0.965 \left( \frac{141.39 \text{ kJ/kg}}{2} \right)
- 1.764 \left( \frac{157.70 \text{ kJ/kg}}{2} \right)
\]
\[
Q_{cu} = 21.1 \text{ kJ}
\]

What do you do if Table A.8 was not available? You could assume specific heat was constant and evaluate the energy equation using
\[
\Delta U = C_v \Delta T, \quad \Delta h = C_p \Delta T
\]

What is the drawback of doing this?