PROBLEM 6.83

A 1 m³, 40 kg Rigid STEEL TANK CONTAINS AIR AT 500 kPa, and BOTH TANK AND AIR ARE AT 20°C. THE TANK IS CONNECTED TO A LINE FLOWING AIR AT 2 MPa and 20°C. THE Valve IS OPENED, ALLOWING AIR TO FLOW INTO THE TANK UNTIL THE PRESSURE REACHES 1.5 MPa, AND THEN IS CLOSED. ASSUME THE AIR AND TANK ARE ALWAYS AT THE SAME TEMPERATURE, AND THE FINAL TEMPERATURE IS 35°C. FIND THE FINAL AIR MASS AND HEAT TRANSFER.

ASSUMPTIONS

1) RIGID TANK
2) NEGLIGIBLE ΔKE
3) NEGLIGIBLE ΔPE
4) ONE INLET/NO EXIT
5) NO SHAFT WORK
6) UNIFORM FLOW PROPERTIES
7) CAN THE AIR BE CONSIDERED IDEAL?

\[
\frac{T_f}{T_{line}} = 2.32
\]

\[
\frac{T_r}{T_{air, initial}} = 2.32
\]

\[
\frac{P_r}{P_{line}} = 0.59
\]

\[
\frac{P_r}{P_{air, initial}} = 0.147
\]

\[Q = \text{From Figure D.1, in both cases, } z = 1 \text{ AND THE AIR IS IDEAL.}\]

SOLUTION

THE ENERGY EQUATION FOR THE TRANSIENT PROCESS IS EXPRESSED AS:
PROBLEM 6.83 - CONT'D

\[ Q_{cv} + \sum m_i (h_i + \frac{V_i^2}{2} + g_z z_i) = \sum m_i (h_0 + \frac{V_0^2}{2} + g_z z_0) \]

So we are left with

\[ Q_{cv} + m_1 h_i \bigg|_{\text{line}} = (m_2 u_2 - m_1 u_1) \bigg|_{\text{exit}}. \quad (1) \]

Note that equation (1) has been derived for a control volume encompassing the steel tank and its contents. So,

\[ Q_{cv} + m_1 h_i \bigg|_{\text{line}} = \]

\[ m_2 u_2 \bigg|_{\text{steel}} + m_2 u_2 \bigg|_{\text{air}} - m_1 u_1 \bigg|_{\text{steel}} - m_1 u_1 \bigg|_{\text{air}} \]

Since the air is ideal

\[ m_1 \bigg|_{\text{air}} = \frac{P_1 V_1}{RT_1} = \frac{(5 \times 10^5 \text{ Pa})(1 \text{ m}^3)}{(8314.5 \text{ J/mol K})(293 \text{ K})} = 5.943 \text{ kg} \]

\[ m_2 \bigg|_{\text{air}} = \frac{P_2 V_2}{RT_2} = \frac{(1.5 \times 10^6 \text{ Pa})(2 \text{ m}^3)}{(8314.5 \text{ J/mol K})(308 \text{ K})} = 16.961 \text{ kg} \]

Now, using continuity

\[ (m_2 - m_1)_{cv} = m_1 \]

\[ 16.961 \text{ kg} - 5.943 \text{ kg} = 11.017 \text{ kg} \]

Since the air is ideal, we can find the internal energies using Table A.7.1.
PROBLEM 6.83- CONT'D

\[ u_2 |_{A_1} = 220.22 \text{ kJ/kg} \]
\[ u_1 |_{A_1} = 209.45 \text{ kJ/kg} \]

SINCE STEEL IS INCOMPRESSIBLE

\[ C_p = C \]

AND WE WILL ASSUME \( C \neq C(T) \) = CONSTANT.

FROM TABLE A.3

\[ C_{\text{steel}} = 0.46 \text{ kJ/kg K} \]

SOLVING THE ENERGY EQUATION, WE FIND

\[ Q_{CV} = -11.017 \text{kJ} \left( \frac{293.60 \text{ kJ/kg}}{\text{kJ}} \right) + 40 \text{kJ} \left( \frac{0.46 \text{ kJ}}{\text{kJ K}} \right) \left( 328.15 \text{K} \right) \]
\[ + 16.96 \text{kJ} \left( \frac{220.22 \text{ kJ}}{\text{kJ}} \right) - 40 \text{kJ} \left( \frac{0.46 \text{ kJ}}{\text{kJ K}} \right) \left( 293.15 \text{K} \right) \]
\[ - 5.943 \text{kJ} \left( \frac{209.45 \text{ kJ/kg}}{\text{kJ}} \right) \]

\[ Q_{CV} = -100.2 \text{kJ} \]

THIS SEEMS A LITTLE COUNTER INTUITIVE. WE FIND FOR THE VALUES GIVEN, THAT HEAT IS TRANSFERRED FROM THE C.I.F. TO THE SURROUNDINGS.

JUST FOR THE SAKE OF CURIOSITY, WHAT IF WE ASSUMED THE TANK WAS ADIABATIC, AND WE WANTED TO FIND THE FINAL TEMPERATURE?
IF WE RETURN TO THE ENERGY EQUATION, WE CAN SET Q_{cu} = 0. BUT, RATHER THAN LOOKING UP FINAL INTERNAL ENERGIES AND ENTHALPIES IN A TABLE, WE'LL ASSUME C_p's AND C_v's FOR AIR ARE ESSENTIALLY CONSTANT, AND SOLVE FOR T_2. THEN

\[ m_1 |_{air} = 5.943 \text{ kg} \]
\[ m_2 |_{air} = \left( \frac{1.5 \times 10^{-6} \text{ m}^3}{1 \text{ m}^3} \right) \left( \frac{28.97 \text{ kg}}{\text{kmol}} \right) \left( \frac{8314.5 \text{ J}}{\text{kmol} \cdot \text{K}} \right) \left( T_2 \right) \]
\[ m_2 |_{air} = \frac{5226.41}{T_2} \]
\[ m_1 = \frac{5226.41}{T_2} - 5.943 \text{ kg} \]

USING \[ C_p |_{air} = 1.004 \text{ kJ/kg K} \]
\[ C_v |_{air} = 0.717 \text{ kJ/kg K} \]

WE FIND

\[ (5226.41/T_2 - 5.943) \text{kJ} \left( \frac{1.004 \text{ kJ}}{\text{kJ} \cdot \text{K}} \right) \left( 293.15 \text{K} \right) \]

\[ = 40 \text{ kJ} \left( \frac{0.46 \text{ kJ}}{\text{kJ} \cdot \text{K}} \right) \left( T_2 - 293.15 \right) + \frac{5226.41 \left( 0.717 \text{ kJ} \right)}{T_2} \left( T_2 \right) \]

\[ - 5.943 \text{ kJ} \left( 0.717 \text{ kJ} \right) \left( 293.15 \right) \]

SOLVING,

\[ T_2 \approx 48.8 \degree C \]
PROBLEM 6.84

A 750 L rigid tank, shown below, initially contains water at 250°C, which is 50% liquid and 50% vapor by volume. A valve at the bottom of the tank is opened, and liquid is slowly withdrawn. Heat transfer takes place such that the temperature remains constant. Find the amount of heat transfer required to reach the state where half the initial mass is withdrawn.

ASSUMPTIONS

(1) Rigid tank
(2) Negligible ΔPE, ΔKE
(3) One exit/no inlets
(4) No shaft work

SOLUTION

The energy equation for the transient process is

\[ Q_{cu} + \sum m_i\left(h_i + \frac{V_i^2}{2} + g_2i\right) = \frac{W_{cu}}{1} + \sum m_e\left(h_e + \frac{V_e^2}{2} + g_2e\right) + \sum m_v\left(u_v + \frac{V_v^2}{2} + g_2v\right) - m_1(u_1 + \frac{V_1^2}{2} + g_21) \]

\[ Q_{cu} = m_e(h_e) + m_v(u_v) - m_1(u_1) \]  (1)

The continuity eqn is

\[ \frac{dm_{cu}}{dt} = m_i - m_e \]

\[ (m_2 - m_1)_{cu} = -m_e \]  (2)

Now we need to fully define states 1 and 2. From table B.1b @ 250°C:
PROBLEM 6.84 - CONT'D

\[ \frac{V_f}{V_g} \bigg|_1 = 1 = \frac{m_f V_f}{m_g V_g} \bigg|_1 \]  \hspace{1cm} (3)

But

\[ m_f V_f = \frac{1}{2} V_{\text{TOTAL}} \]  \hspace{1cm} (4)

So substituting eqn (4) into (3) yields

\[ m_f = \frac{1}{2} \frac{V_{\text{TOTAL}}}{V_f \bigg|_1} = \frac{0.375 \text{m}^3}{0.001251 \text{m}^3/\text{kg}} \]

\[ m_f \bigg|_1 = 299.76 \text{ kg} \]

Similarly, we find

\[ m_g \bigg|_1 = \frac{0.375 \text{m}^3}{0.05013 \text{m}^3/\text{kg}} = 7.48 \text{ kg} \]

At state 2 we know

\[ m_2 = \frac{1}{2} m_1 = \frac{1}{2} (m_f \bigg|_1 + m_g \bigg|_1) \]

\[ m_2 = 153.62 \text{ kg} \]

So

\[ V_2 = \frac{0.750 \text{ m}^3}{153.62 \text{ kg}} = 0.004882 \frac{\text{m}^3}{\text{kg}} \]
PROBLEM 6.34 - CONT'D

SINCE THE TEMPERATURE REMAINS CONSTANT
\[ T_2 = T_2 = 250^\circ C \]

FROM TABLE B.1.1 WE SEE THAT THE MIXTURE REMAINS IN THE TWO-PHASE REGION, AND THE QUALITY IN THE TANK AT STATE (2) IS

\[ 0.00488 \, \text{m}^3/\text{kg}_j = (1-x_2)0.001251 \, \text{m}^3 + x_2(0.05013 \, \text{m}^3) \]

\[ x_2 = 0.07429 \]

THEN,

\[ U_2 = (1-x_2)(U_4 \big|_{(1)}) + x_2(U_g \big|_{(2)}) \]

\[ U_2 = (0.9257)(1080.37 \, \frac{\text{kJ}}{\text{kg}}) + 0.07429(2602.37 \, \frac{\text{kJ}}{\text{kg}}) \]

\[ U_2 = 1193.44 \, \frac{\text{kJ}}{\text{kg}} \]

NOW WE CAN EVALUATE EQN. (1)

\[ Q_{cu} = (153.62 \, \text{kJ})(1085.34 \, \frac{\text{kJ}}{\text{kg}}) + (153.62 \, \text{kJ})(1193.44 \, \frac{\text{kJ}}{\text{kg}}) \]

\[ = m_1 U_1 \]

THE VALUE OF \( m_1 U_1 \) CAN BE FOUND BY

\[ m_1 U_1 = 299.76 \, \text{kJ} (1080.37 \, \frac{\text{kJ}}{\text{kg}}) + 7.48 \, \text{kJ} (2602.37 \, \frac{\text{kJ}}{\text{kg}}) \]

\[ m_1 U_1 = 343.31 \, 7.44 \, \text{kJ} \]

OR THE VALUE OF \( U_1 \) CAN BE FOUND

\[ U_1 = (1-x_1)(U_4 \big|_{(0)}) + x_1(U_g \big|_{(0)}) \]

WHERE

\[ x_1 = \frac{m_g \big|_{(0)}}{m_{tot} \big|_{(0)}} = 0.024346 \]
SO \[ U_1 = 1117.42 \text{ kJ/kg} \]

Finally,

\[ Q_{cu} = (153.62 \text{ kJ})(1085.34 \text{ kJ/kg}) + (153.62 \text{ kJ})(119344.4 \text{ kJ/kg}) \]
\[ - (307.24 \text{ kJ})(1117.42 \text{ kJ/kg}) \]

\[ Q_{cu} = 6750.1 \text{ kJ} \]