Day 7 – Example of Indeterminate System – Type 2

(... parallel bars, cylinders, whatever ...)
Strategy: assume $P$ is applied across some plate that adds stress to each component. We'll note that both materials have the same $f$. We'll relate the $f$'s to the forces in each using $\frac{PL}{AE}$, and we'll also relate the forces, say, using a FBD of the (assumed) plate... namely $P = P_s + P_c$.

Solution:
\[
\delta_P = \delta_c = \delta_s = ?
\]
\[
\delta_c = \frac{P_c L_c}{A_c E_c}
\]
\[
\delta_s = \frac{P_s L_s}{A_s E_s}
\]
$L_S = L_C = 12 \text{ in}$

$A_S = \frac{\pi (4)^2}{4} = 12.57 \text{ in}^2$

$A_C = \frac{\pi (4)^2}{4} - 12.57 = 63.6 - 12.6 = 51.0 \text{ in}^2$

$E_C = \text{Let's use } 17,600 \text{ ksi (see p. 913)}$

$E_S = 29 \times 10^6 \text{ psi, memorized}$

$$\frac{L_C}{A_C E_C} = \frac{12 \text{ in}}{51.0 \text{ in}^2} \times \frac{17,600 \times 10^3 \text{ lb}}{17 \times 10^6 \text{ lb}} = 1.384 \times 10^{-8} \text{ in}$$

$$\frac{L_S}{A_S E_S} = \frac{12 \text{ in}}{12.6 \text{ in}^2} \times \frac{29 \times 10^6 \text{ lb}}{29 \times 10^6 \text{ lb}} = 3.284 \times 10^{-8} \text{ in}$$

So... $\delta = \delta_c = \delta_S$

$$P_C \times 1.384 \times 10^{-8} \text{ in} = P_S \times 3.284 \times 10^{-8} \text{ in}$$

or

$$P_C = \frac{P_S \times 3.284 \times 10^{-8}}{1.384 \times 10^{-8}} = 2.372 \ P_S$$
now...

\( \varepsilon F_y = 0 \) across the bearing plate

\[-P + P_s + P_c = 0\]

\[P = P_s + P_c\]

Substituting from p. 2

\[P = P_s + 2.372 P_s = 3.372 P_s\]

\[P = 25 \text{k}\]

\[P_s = \frac{25 \text{k}}{3.372} = 7412.16 \text{ lb}\]

\[P_c = 2.372 P_s = 17,582 \text{ lb}\]

Check: \[17,582 + 714 = ? \text{ 25k}\] \( \checkmark \)

So, the copper takes 17,600 lb

and the steel takes 7400 lb.

Now for g...
\[ \delta = \frac{P_0 L_c}{A c E_c} = \frac{P S L_s}{A_S E_S} \]

Let's use copper

\[ \delta = \frac{17,582 \text{ lb} \, \text{(12 in)}}{51.0 \text{ in}^2 \times 17 \times 10^6 \text{ lb}} = 0.00024 \text{ in} \]

Answer:
- Load in copper: 17.6 k
- Load in steel: 7.9 k
- \( \delta \) under 25 k is 0.00024 in

Discussion: Let's check. Filler said we can add the stiffnesses in a parallel system.

\[ K_1 = \frac{1}{1.384 \times 10^{-8} \text{ in/1lb}} = 72.25 \times 10^6 \frac{\text{lb}}{\text{in}} \]

\[ K_2 = \frac{1}{3.284 \times 10^{-8} \text{ in/1lb}} = 30.45 \times 10^6 \frac{\text{lb}}{\text{in}} \]

Add... \[ 102.7 \times 10^6 \frac{\text{lb}}{\text{in}} \]

\[ P = K \delta \quad \delta = \frac{P}{K} = \frac{25,000 \text{ lb}}{102.7 \times 10^6 \text{ lb/in}} = 0.00024 \text{ in} \]
YEAH!