1. Okay, let's pick up from where we left off on that one other example ... the one we looked at during the 'Review' for the last exam. Beam spans 20 ft. Ttrib width at one end is \( \frac{1}{2} \) of 12 ft on one side and zero on the other ... and at the other end is \( \frac{1}{2} \) of 12 ft on one side and \( \frac{1}{2} \) of 8 ft on the other. So, the area is 'trapezoidal' shape, and thus also the distributed (line) load. Go ahead and include the sketch from your notes that day. The area loads for the problem are 12 psf DL and 50 psf LL. And, as we did in the review, let's use/assume 20 plf self weight for the beam.

We'll use a beam that is 5-1/8 wide by 15.5 in. deep, and loaded in the strong direction. The beam \( E = 1.8 \times 10^6 \) psi. Allowable bending stress is 2400 psi; allowable shear stress is 240 psi. The allowable deflection is \( L/360 \) for Live load and \( L/240 \) for Total Load.

Find:

a) check bending stress at position of max moment.

b) check shear stress at location `d` from the end where the shear will be max.

c) check Live load deflection (by method of intergration of the load)

d) check Total load deflection using a long-term DL deflection of 1.5 x the immediate DL deflection.

e) if all the checks are okay, then check the assumption of 20 plf for self weight, assuming the wood weighs 35 pcf. If any of the design checks don't work out – let's get a beam size that does work, by increasing the depth some multiple of 1.5 in. ... and note that every time we increase the beam size, we increase the self weight.

from lecture...
Let's check the R × N.S.

\[ W_1 \]

\[
\begin{array}{c}
\frac{640}{-392} \\
\hline
\frac{248}{392}
\end{array}
\]

\[ W_1 = \frac{1}{2}(20) \times 248 \]

\[ W_1 = 248 \times 2 = 496 \]

\[ W_2 = 20 \times 392 = 7840 \]

\[ V \leq M_{R1} = 0 \Rightarrow R_2 \]

\[ 2480(13,33) + 7840(10) - R_2(20) = 0 \]

\[ R_2 = 5573 + 16 \]

\[ V \leq M_{R2} = 0 \Rightarrow R_1 \]

\[ R_1(20) - 2480(6.67) - 7840(10) = 0 \]

\[ R_1 = 4747 + 16 \]

Good
Let's get an expression for \( w(x) \):

\[
w(x) = 392 + \frac{640-392}{20} x = 392 + 12.4 x
\]

Check:
- Let \( x=0 \) ... \( w(0) = 392 \)
- Let \( x=20 \) ... \( w(20) = 392 + 12.4(20) = 640 \)

\[
\boxed{w(x) = 392 + 12.4 x}
\]

Integrate to get \( V(x) \):

\[-w = \frac{dV}{dx}\]

\[
dV = -w(x) \, dx
\]

\[
V = \int -w(x) \, dx + C
\]

\[
V(x) = \int (-392 - 12.4 x) \, dx + C
\]

\[
V(x) = -392x - 12.4 \frac{x^2}{2} + C
\]

\[
V(x) = -392x - 6.2 x^2 + C
\]

Let \( x=0 \) ... \( V = 4747 \)

\[
4747 = -392(0) - 6.2 (0)^2 + C
\]

\[
C = 4747
\]
check ... let \( x = 20 \)

\[
V(20) = 4747 - 392(20) - 6.2(20)^2 =
\]

\[
= -5573 ...
\]

yes! That's the R&M at the other end...

So...

\[
V(x) = 4747 - 392x - 6.2x^2
\]

Let's integrate to get \( M \)...

\[
\frac{dM}{dx} = V
\]

\[
dM = Vdx
\]

\[
M = \int Vdx + C
\]

\[
= 4747x - 392x^2 - 6.2x^3 + C
\]

\[
M(x) = 4747x - 196x^2 - 2.067x^3 + C
\]

Let \( x = 0 \) ... \( M = 0 \) solve for \( C \)

gives \( C = 0 \)

\[
M(x) = 4747x - 196x^2 - 2.067x^3
\]

check ... \( M(20) = ? \).
474.7(20) - 196(20)^2 - 2.067(20)^3 = 0 \checkmark \rho_{5} \text{ good}

\[ M(x) = 474.7x - 196x^2 - 2.067x^3 \]

Let's plot this up... find what the max moment is... Note... this is TOTAL LOAD... 

Looks like \( M \) is max at \( x=10.4 \) ft. 

\[ M_{\text{max}} = 25,845 \text{ lb-ft} = \frac{310,140 \text{ lb-in}}{232.55 \text{ in}^3} \]

So... for part a)...

\[ f_b = \frac{M}{S} \quad S = \frac{kh^2}{6} = \frac{(5.125)(16.5)^2}{6} = 232.55 \text{ in}^3 \]

\[ f_b = \frac{310,140 \text{ lb-in}}{232.55 \text{ in}^3} = 1334 \text{ psi} \]

\( f_b = 1334 \leq f_b' = 2400 \text{ psi? yes} \)

or \( \frac{f_b}{f_b'} = \frac{1334}{2400} = 0.556 \leq 1.00 \text{ good} \ checkmark \)
Now let's tackle shear...

"d" from each end is 16.5 in or

\[ \frac{16.5}{12} = 1.375 \, \text{ft} \]

Let's dump \( x = 1.375 \) and \( x = \frac{-1.375}{18.625} \)

into our expression for shear...

\[
V(1.375) = 47.47 - 39.2(1.375) - 6.2(1.375)^2 = \frac{419.6}{16}
\]

\[
V(18.625) = \frac{-470.5}{16} \}
\]

\text{actually... I pulled these off my spreadsheet}

So...

actually, we only need the second one...

\[
\max f_v = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{470.5}{5,125(16.5)} \text{ in}^2 = \frac{83 \, \text{psi}}{}
\]

\[ V_{\@ d} = 470.5 \, \text{lb} \]

\[
\text{Is } f_v = 83 \leq F_v = 240 \, \text{? yes! good} \]

\[
\frac{83}{240} = 0.345 \leq 1.00 \, \text{ in terms of unity check.}
\]
Whoa, now things get a bit ugly ... since now we have to break out LL and DL ...

Well, going back to our load diagram ... and looking at LIVE LOAD ...

Starts with 300 plf and ends with 500 plf ...

Our expression for w is ... \( w(x) = 300 + \{(500-300)/20\} x \) ... = ... 300 + 10 x

So, \( w(x) = 300 + 10 x \)

We need the reactions ... using the sketches from before, but with different numbers ...

\( W_1 = \frac{1}{2} (20)(200) = 2000 \text{ lb} \)

\( W_2 = 300 \times 20 = 6000 \text{ lb} \)

\( R_2 = [2000 (13.33) + 6000 (10)] / 20 = 4333 \text{ lb} \)

\( R_1 = [2000 (6.67) + 6000 (10)] / 20 = 3667 \text{ lb} \)

Check ... 4333 + 3667 =? 2000 + 6000 ... yes.

So ...

\( V = \int -w \ dx + C = \int (300 + 10 x) \ dx + C = -300 x - 10 x^2 / 2 + C ... \)

At \( x = 0 \) we know that \( V = 3667 \) ... so ... \( C = 3667 \) ...

\( V(x) = 3667 - 300x - 5 x^2 \)

Check ... \( V(20) = -4333 \) ... good.

\( M = \int V \ dx + C = \int (3667 - 300 x - 5 x^2) \ dx + C \)

\( M = 3667 x - 300 x^2 / 2 - 5 x^3 / 3 + C ... \)

Dumping in \( M = 0 \) at \( x = 0 \) gives \( C = 0 ... \)

\( M(x) = 3667 x - 150 x^2 - 1.667 x^3 ... \)

Check ... \( M @ x = 20 = ... 0 ... \) good.

Now ... \( M = EI \ \theta' \)

So, \( EI \ \theta = \int M \ dx + C \)
\[= 3667 x^2 / 2 - 150 x^3 / 3 - 1.6667 x^4 / 4 + C =\]

\[EI \theta = 1833 x^2 - 50 x^3 - .4166 x^4 + C\]

... and we can't easily get rid of this C ... so let's keep going ...

\[EI v = \int EI \theta \, dx + C_2\]

\[= 1833 x^3 / 3 - 50 x^4 / 4 - .4167 x^5 / 5 + C x + C_2\]

\[= 611 x^3 - 12.5 x^4 - .0833 x^5 + C x + C_2\]

\[EI v \text{ has to be zero at zero } \ldots \text{ so } C_2 = 0 \ldots\]

\[EI v \text{ has to be zero at } x = 20 \ldots \text{ should let us solve for } C \ldots\]

\[0 = 611 \times 20^3 - 12.5 \times 20^4 - .0833 \times 20^5 + C \times 20\]

\[C = -131,000 \ldots\]

So,

\[EI v = 611 x^3 - 12.5 x^4 - .0833 x^5 - 131,000 x \ldots\]

... we get a max value of about \(-832,400\) at \(x = 10.1 \text{ ft} \ldots\)

So, For Live load ...

\[EI v \text{ max } = -832,400 \text{ lb-ft}^3 \ldots\]

\[v \text{ max } = 832,400 \text{ lb-ft}^3 / EI\]

\[E = 1,800,000 \text{ psi}\]

\[I = bh^3 / 12 = 5.125 (16.5)^3 / 12 = 1918.5 \text{ in.}^4 \ldots\]

\[v \text{ max } = \Delta_{ll} = 832,400 \text{ lb-ft}^3 \text{ in.}^2 (12 \text{ in./ft})^3 / (1,800,000 \text{ lb } * 1918.5 \text{ in.}^4) = \]

\[\ldots 0.4165 \text{ in.}\]

WHOA! The last bit was a lot of work!

Check against limit ...

Is 0.42 in. \(\leq L/360 = 20 (12)/360 = 0.667 \text{ in.}\)

Yes, good.
Now, for DL Deflection...

one end: \( w = 20 + 72 = \frac{92}{2} \text{ ft} \)

other end: \( w = 20 + 120 = 140 \text{ ft} \)

\[ W = 92 + \frac{(140 - 92)}{20} x = 92 + 2.40 x \]

\[ V = -\int w(x) \, dx + C \]

\[ = -92x - 2.4\frac{x^2}{2} + C \]

\[ = -92x - 1.2x^2 + C \]

To get C we need to solve for the

\[ WM \]

\[ W_1 = \frac{1}{2} (48) 20 = \frac{480}{2} \]

\[ W_2 = 92(20) = 1840 \text{ ft} \]

\[ 2 \Sigma M_{k_1} = 0 \Rightarrow R_2 \]

\[ 480(13.33) + 1840(10) - R_2(20) = \]

\[ 0 \]
\[ R_2 = 1240 \quad 16 \uparrow \]

\[ 2 \leq M_{22} = 20 \Rightarrow R_1 = \frac{4.67(480) + 10(1840)}{20} \\
= 1080 \quad 16 \uparrow \]

**Check** is \( 1240 + 1080 = 480 + 1840 \)?

*Yes!*

So...

\[ V(x) = 1080 - 92x - 1.2x^2 \quad (DL + SN) \]

\[ M = \int V(x) \, dx + c \]

\[ = 1080x - \frac{92}{2}x^2 - \frac{1.2}{3}x^3 + c \]

Let \( M = 0 \) at \( x = 0 \) gives \( c = 0 \)

\[ M(x) = 1080x - 46x^2 - 0.402x^3 \]

**Check**...

Let \( x = 20 \) --

\[ 1080(20) - 46(20)^2 - 0.402(20)^3 = 0 \quad \checkmark \]
\[ EI\theta = \int M(x) \, dx + C \]
\[ = \frac{1080}{2} x^2 - \frac{46}{3} x^3 - \frac{0.40}{4} x^4 + C \]
\[ EI\theta = \frac{540}{3} x^2 - 15.333 x^3 - 0.100 x^4 + C \]
and we'll have to carry the \( C \) along...

\[ EI\nu = \int EI\theta(x) \, dx + C_2 \]

\[ EI\nu = \frac{540}{3} x^3 - \frac{15.333}{4} x^4 - \frac{0.100}{5} x^5 + Cx + C_2 \]

\[ EI\nu = 180 x^3 - 3.833 x^4 - 0.020 x^5 + Cx + C_2 \]
non \( \nu = 0 \) at \( x = 0 \) so \( C_2 = 0 \)
and \( \nu = 0 \) at \( x = 20 \), so...

\[ 180 (20)^3 - 3.833 (20)^4 - 0.020 (20)^5 + C (20) = 0 \]

\[ C = -38,136 \]

so...

\[ EI\nu = 180 x^3 - 3.833 x^4 - 0.020 x^5 - 38,136 x \]

Let's plot it and find max defl.
looks like we get a max of about 241,708 = EIV \_\text{max} \text{ at about } \chi = 10, 1...

Also... this is for immediate or deflection...
for the long term effect... it all gets multiplied by 1.5...

\[
EIV_{\text{long term}} = 1.5 \left( 180 \chi^3 - 3.833 \chi^4 - 0.020 \chi^5 - 381.36 \chi \right)
\]
\[
= 270 \chi^3 - 5.75 \chi^4 - 0.030 \chi^5 - 57,200 \chi
\]

Now... let's add this to the live load deflection...
from p.8

\[
EIV = 611 \chi^3 - 12.5 \chi^4 - 0.0833 \chi^5 - 131,000 \chi
\]

Add...

\[
881 \chi^3 - 18.25 \chi^4 - 0.1133 \chi^5 - 188,200
\]

\[
EIV_{\text{LL+1.5DL}}
\]

spreadsheet... This value is \text{max} at about...
- 1,195,000 = \text{EI} \varepsilon \max \text{ @ about 10.1 ft}

so...

\[ \frac{V_{\max} \mu + 1.5 \mu L}{2} = \frac{1195,000 \text{ lb}-\text{ft}^3 (12 \text{ in/ft})^3}{1.8 \times 10^6 \text{ 1918.5 in}^4} \]

\[ V_{\max} \mu + 1.5 \mu L = 0.598 \text{ in} \]

is this less than or equal to L/240?

\[ L/240 = \frac{20 \times 12}{240} = 1.00 \text{ in}. \]

is 0.60 in \leq 1.00 in? ...Yes...Good!

Okay bending good
shear good
LL Deflection good
TL deflection including creep, good

S.W. \[ w_{\text{s.w.}} = \frac{5.125 \times 16.5 \times 35}{12} \]

Close enough! We won't gonna redo all this for 0.55 plf.
Summary...

Part ... a) $fb = 1330 \text{ psi} \leq 2400 \text{ psi}$ ... good ... (my page 5)

   b) $fv \text{ @ distance } d = 83 \text{ psi} \leq 240 \text{ psi}$ ... good ... (my page 6)

   c) $\Delta LL = 0.42 \text{ in.} \leq L/360 = 0.67 \text{ in.}$ ... good ... (my page 8)

   d) $\Delta LL + 1.5 \text{ DL+SW} = 0.60 \text{ in.} \leq L/240 = 1.00 \text{ in.}$ ... good (my page 13) ...

   e) Self weight = 20.55 plf ... not quite $\leq 20$ used in all the calcs ... but I'm not going to redo it for .55 plf.

So ... good ... done!

But, obviously, the next size up would also work.
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<th>V(x)</th>
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<td>10.35</td>
<td>25,844</td>
<td>25.64</td>
</tr>
</tbody>
</table>

1.375  | 4196.28 |
18.625  | -4704.72 |
LL + 1.5 DL deflection ...

\[
\begin{pmatrix}
\varepsilon \\
-1 \\

\end{pmatrix}
\]

|   | 0   | 0   | 1  | -187,337 | 2  | -369,648 | 3  | -542,319 | 4  | -701,204 | 5  | -842,635 | 6  | -963,437 | 7  | -1,060,939 | 8  | -1,132,993 | 9  | -1,177,980 | 10 | -1,194,830 | 11 | -1,183,034 | 12 | -1,142,657 | 13 | -1,074,349 | 14 | -979,363 | 15 | -859,568 | 16 | -717,460 | 17 | -556,175 | 18 | -379,508 | 19 | -191,921 | 20 | 1,440 |
|---|-----|-----|----|----------|----|----------|----|----------|----|----------|----|----------|----|----------|----|----------|----|----------|----|----------|----|----------|----|----------|----|----------|----|----------|----|----------|
| 10.1 | -1,194,943 |
| 10.2 | -1,194,769 |
| 10.3 | -1,194,307 |
| 10.4 | -1,193,559 |