Assignment 15 ... Solution

1. Composite MOI with (1) # 4 ... section not cracked.

Assuming again what we did in class, together, is correct (scary thought) ...

For the un-cracked section ...

\[ y_{\text{bar}} = 3.86 \text{ in. (down from the top)} \]

\[ A = 5 \times 24 + 4 \times 8 = 152 \text{ in.}^2 \]

and

\[ I_g = 1488 \text{ in.}^4 \text{ (with respect to the n.a.)} \]

By transforming the steel rebar to concrete, we end up adding \( (n-1) \) \( A_s = (9.4 - 1) (0.20 \text{ in.}^2) = 1.68 \text{ in.}^2 \).

Note: the area of a single #4 bar is a bit less than 0.20, but I'll use 0.20 on through.

So, let's calc a new Area, and n.a., and new MOI (w.r.t the n.a.).

\[ y_{\text{bar}} (152 + 1.68) = 152 (3.86) + (5 + 8 - 2) 1.68 \ldots \]

\[ y_{\text{bar}} = 3.94 \text{ in.} \]

... yeah, it sank a bit ...

Now for the MOI ...

\[ \text{MOI}_{\text{new n.a.}} = \text{Sum of the MOIs of the pieces w.r.t this new n.a. = ...} \]

\[ \text{MOI}_{\text{new n.a.}} = 1488 + 152 (3.94 - 3.86) \times 2 + 0 + 1.68 (11 - 3.94) \times 2 \ldots \]

\[ = 1488 + 1.0 + 0 + 83.7 = 1573 \text{ in.}^4 \]
Yeah! (I suppose.)

Okay, now to deal with the “0” ... (Extra Credit)

Note: “0” is the MOI of the n-1 round ‘concrete bars' w.r.t their own centroidal axis ... I assumed the number to be small ... let's see.

From the back of the book, the MOI of a circular section about an x-x axis through the center is ... \( \pi d^4 / 64 = 3.14 (0.5)^4 / 64 = 0.003. \)

If there are n-1 = 8.4 of them, then ... \( 8.4 \times 0.004 = 0.026 \text{ in.}^4 \) ...

Doesn't even show up in the other numbers we are using.

**Answer:** I_uncracked, incl rebar, transformed = 1573 \text{ in.}^4 ...

**Compare / Contrast with Ig:** ... 1573 vs 1488 ...

\[ 1573 / 1488 = 1.057 \] ...

Or, alternately, using Ig only underestimates the section MOI by 5%.

I suspect the writers of the Code recognized that for all the work of including the rebar in the calc ... it doesn't change things much. Also, pretty early on, the sections are going to crack, and so there is really not very much use for the uncracked MOI.

2. **Cracked MOI** ...

... we did this in class ... I\_cracked section ... \( 194 \text{ in.}^4 \) ... \( y_{\text{bar}} = 1.24 \text{ in.} \) down from the top.

... the n.a climbed ... The MOI was hugely reduced.

But, the beam was made stronger, as we shall see.

**Answer:** I\_cracked section ... \( 194 \text{ in.}^4 \)
3. **Extreme fiber compressive stress under the load of the self weight only** ...

\[ \sigma_{\text{extreme fiber, compression, s.w.}} = \frac{M_{\text{s.w.}} (y_c)}{I_{\text{cracked}}} = \ldots \]

\[ \sigma = 6400 \text{ (12) lb-in. (1.24 in.) / 194 in.}^4 = 491 \text{ psi.} \]

The concrete has taken on more compressive stress because we have less section due to the `cracked' condition.

But it is still way less than the 3000 psi `strength' of the concrete. Good.

**Answer:** \( \sigma_{\text{extreme fiber, compression, s.w.}} = 490 \text{ psi.} \)

4. **Extreme fiber tensile stress in the steel** ... (under the load of self weight only).

Now we use \( y_t \) ...

\[ y_t = 11 - 1.24 = 9.76 \text{ in.} \]

\[ \sigma = 6400 \text{ (12) 9.76 / 194 = 3864 psi ... call it 3860 psi.} \]

BUT, this is in the steel transformed to concrete. We need to see what the actual steel feels. Well, this 3860 psi will actually be concentrated by the ratio `n' ...

\[ \sigma_{\text{steel}} = 3860 \text{ psi (9.4) = 36,300 psi} \ldots \text{ or 36.3 ksi.} \]

**Answer:** under the self weight only, the extreme fiber stress in the steel is 36.3 ksi. (36,300 psi)

Now, the good news is that this is significantly less than the yield stress of the steel, so at least it's not coming down on us.
5. The Total Applied Moment corresponding to a stress in the steel of 60,000 psi.

Assuming linear behavior, the total applied moment will be ... either you can believe me on this, or crank it out from repeating the above procedures with the new extreme fiber stress for steel ...

\[ M = 6400 \text{ lb-ft} \left( \frac{60,000}{36,300} \right) = 6400 \text{ lb-ft} \times 1.65 = 10,570 \text{ lb-ft} \ldots \text{or} \ldots 127,000 \text{ lb-in.} \]

**Answer:** 10,600 lb-ft or 127,000 lb-in.

**Extra Credit:** ... the extreme fiber compressive stress at the above condition (no. 5).

\[ \sigma_{\text{max, comp}} = 490 \text{ psi} \times 1.65 = 809 \text{ psi.} \]

That is still well below the compressive strength of the concrete. Good. It also means our `linear elastic' analysis is providing good numbers.