Day 7, Assignment

1. Go back and look at Assignment 6, Problem 1 ... the problem where we had a couple links loaded with a force $P = 10k$ that was oriented down and to the right, you were to calc the deflection of point $P$, and so on. Now take that same problem and (without the 10k acting on it) ... impose a displacement of $P$ exactly 0.0055 in. straight to the right. Now, determine: a) the force applied to $P$ that would be necessary to cause such a displacement (mag and direction), and b) the corresponding stiffness of the system with respect to such a displacement. Discuss: is your answer similar or dissimilar to Assn 6 Prob 1???

2. 2.4-9 ... except don't do part a) (unless you really want to) ... do do part b) ... and let's add a part c), namely, if $L = 6$ in., find the displacement of the plate (load point). Just glancing at this - I would assume that $P$ is the whole force, even though there are actually two $P$ each with an arrow.

3. 2-4-15 ... this problem has a ** ... I hope it isn't too hard.
Okay, let's look at the elongation of each piece...

\[ s_{AB} = 0.0055 \text{ in} \]

\[ \text{piece } AB \]

\[ \text{AB just elongates the } \]
\[ 0.0055 \text{ in} \]

\[ \text{piece } BC \ldots \text{ and B swings} \]

\[ s_{BC} = 0.0055 \text{ in} \times \sin 30^\circ \]
\[ = 0.00275 \text{ in} \]

Now let's calc the P's necessary to do that...
0.0055 in $= \frac{P_{AB}}{9.3 \times (18 \text{ in}) \frac{\text{in}^2}{2.25 \text{ in}^2} (29 \times 10^6 \text{ lb}) \frac{\text{in}^2}{1 \text{ in}^2}}$

Earlier Solution

\[ P_{AB} = \frac{0.0055 \text{ in}}{18 \text{ in}} \frac{2.25 \text{ in}^2}{1 \text{ in}^2} 29 \times 10^6 \text{ lb} = \]

\[ = \frac{19.938 \text{ lb}}{} \rightarrow \]

Similarly

\[ P_{BC} \]

\[ 0.00275 \text{ in} = \frac{P_{BC}}{36 \text{ in}} \frac{\text{in}^2}{3.14 \text{ in}^2} (29 \times 10^6 \text{ lb}) \frac{\text{in}^2}{1 \text{ in}^2} \]

\[ P_{BC} = \frac{0.00275 \times (3.14) (29 \times 10^6 \text{ lb})}{36} \]

\[ = 6956 \text{ lb} \uparrow \]

Now let's add the P's.
\[ \varepsilon F_x = P_x \quad \vdots \]
\[ 19.94 \text{k} + 6.96 \cos 60^\circ = 23.4 \text{k} = \varepsilon F_x \]
\[ \varepsilon F_y = P_y \]
\[ 6.96 \text{k} \sin 60^\circ = 6.03 \text{k} = P_y \]
\[ P = \sqrt{(23.4)^2 + (6.03)^2} = 24.2 \text{k} \]
\[ \theta = \tan^{-1} \frac{6.03}{23.4} = 14.4^\circ \]
\[ \rho = 24.2 \text{k} \angle 14^\circ \]

**Stiffness**
\[ k = \frac{P}{\varepsilon} = \frac{24.2 \text{k}}{0.0055 \text{in}} = 4400 \text{ lb/in} \]

In this orientation, the stiffness is different than the assignment 6 situation.
2.4-9 Find stresses in the steel and aluminum, and, also, $S$ for $L = 6 \text{ in}$...

\[ P = 12 \text{k} \]

\[ A_{\text{steel}} = 1.03 \text{ in}^2 \]
\[ A_{\text{alum}} = 8.92 \text{ in}^2 \]
\[ E_{\text{alum}} = 10 \times 10^6 \text{ psi} \]
\[ E_{\text{steel}} = 29 \times 10^6 \text{ psi} \]

Strategy: To move the load point, we will have to flange the steel and compress the aluminum. This is a parallel bar problem... stiffness of system equals sum of stiffnesses of each.

Solution: Let's do it...

\[ K_{\text{alum}} = \frac{AE}{L} = \frac{8.92 \text{ in}^2 \times 10 \times 10^6 \text{ psi}}{12 \text{ in}} = 7,433,333 \frac{\text{lb}}{\text{in}} \]
\[ k_{steel} = \frac{1.63 \times 10^6}{6} = 497833.3 \text{ lb/in} \]

\[ k_{equiv} = k_{al} + k_{st} = 12,411,667 \text{ lb/in} \]

and now...

\[ P = k_5 \]
\[ S = \frac{P}{k} = \frac{12,411,667 \text{ lb}}{12,411,667 \text{ lb/in}} = 1 \text{ in} \]

\[ S = 0.00097 \text{ in} \]

Now let's call the stresses in each... Load goes to stiffness...

So...

\[ P_{alum} = \frac{P_{total}}{k_{equiv}} \times k_{alum} = 12,120 \text{ lb} \times \frac{7.433}{12,411} = 7.187 \text{ k} \]

\[ P_{alum} = 7.187 \text{ k} \]

\[ \sigma_{alum} = \frac{P}{A} = \frac{7.187 \text{ k}}{8.92 \text{ in}^2} = 806 \text{ psi} \]

\[ P_{steel} = P - P_{alum} = 12 \text{ k} - 7.187 \text{ k} = 4.813 \text{ k} \]

\[ \sigma_{steel} = \frac{4.813}{1.03} = 4.673 \text{ ksi} = 4673 \text{ psi} \]
\[ \sigma_{\text{alum}} = 806 \text{ psi} \quad (810) \]

\[ \sigma_{\text{steel}} = 4670 \text{ psi} \quad (4700 \text{ psi}) \]

ops. gang! I read the problem wrong... I assumed ONE P. ... and there are TWO... But, no problem... we'll assume we're in the linear elastic range...

\[ \delta = 2 \times 0.00047 \text{ in} = 0.00094 \text{ in} \]

\[ \sigma_{\text{alum}} = 2 \times 806 = 1612 = 1610 \text{ psi} \quad \text{Compression} \]

\[ \sigma_{\text{steel}} = 2 \times 4670 = 9340 = 9340 \text{ psi} \quad \text{Tension} \]

Now, let's check...

\[ P_{\text{steel}} = 2 \times 4.813 = 9.626 \text{ lb} \]

\[ \delta_{\text{steel}} = \frac{P_l}{AE} = \frac{9626 \text{ (6 in)}}{1.03 \text{ in}^2 \times 29 \times 10^6 \text{ lb}} = \]

\[ = 0.00193 \text{ in} \]

Yeah!
Given: Rigid bar with wires with $E = 30 \times 10^6$ psi... and $A = .0272$ in$^2$ each.  

Find: a) Stresses in wires and  
     b) displacement of end B

**Strategy:** Let's relate the $f$'s via geometry...

Then relate the forces to one another via the geometry and stiffness of the wires...  

Then a moment equation to relate the forces...

And hopefully solve...

**Solution**

**Geometry**

\[ \delta_P = \frac{50}{20} \delta_C = 2.5 \delta_C \]

\[ \delta_{13} = \frac{60}{20} \delta_C = 3.3 \delta_C \]
\[ \delta_c = \frac{P_c L_c}{A_c E_c} \]

\[ \delta_c = \frac{P_c \left( 18.1^2 \right)}{0.0272 \frac{1}{\text{in}^2} \times 30 \times 10^6 \frac{\text{lb}}{\text{in}^2}} = \frac{P_c 0.000022 \text{ in}}{15} \]

\[ \delta_d = \frac{P_d L_d}{A_d E_d} = \frac{P_d 36 \text{ in}}{0.0272 \frac{1}{\text{in}^2} \times 30 \times 10^6 \frac{\text{lb}}{\text{in}^2}} \]

\[ \delta_d = \frac{P_d 0.000044 \text{ in}}{15} \]

Let's relate one force in terms of the other through the \( \delta \)'s:

\[ \delta_d = 2.5 \delta_c \]

\[ P_d 0.000044 \frac{\text{in}}{15} = 2.5 \left( 0.000022 \frac{P_c}{\text{in}} \right) \]

\[ P_d = P_c 2.5 \left( \frac{22}{44} \right) = 1.25 P_c \]
Now let's take a moment Equation

\[ 3 \frac{M}{A} = 0 \]

\[ \begin{align*}
\epsilon &= 0 \\
Ax &= 0 \\
Ay &= 0 \\
P_c &= 1.25 P_c \\
340 &= 13
\end{align*} \]

\[ -20 P_c - 50 (1.25 P_c) + 340 (66) = 0 \]

\[ P_c = \frac{340 \times 66}{20 + 1.25 \times 50} \text{ lb/in} \]

\[ P_c = 272 \text{ lb} \]

\[ P_D = 1.25 P_c = 1.25 \times 272 = 340 \text{ lb} \]

**Stress**

\[ \sigma_c = \frac{272 \text{ lb}}{0.0272 \text{ in}^2} = 10,000 \text{ psi} \]

\[ \sigma_D = \frac{340 \text{ lb}}{0.0272} = 12,500 \text{ psi} \]

and now for \( \delta_B \)...
\[ \delta_c = P_c \left( 0.00022 \left( \frac{\text{in}}{15} \right) \right) \]
\[ = 272 \text{ lb} \times 0.00022 \left( \frac{\text{in}}{15} \right) = \]
\[ = 0.005984 \text{ in} \]

\[ \delta_{13} = 3.3 \delta_c = 3.3 \times 0.005984 = \]
\[ \delta_{13} = 0.0197 \text{ in} \]

**Answers.**

\[ \sigma_c = 10,000 \text{ psi, Tension} \]
\[ \sigma_{13} = 12,500 \text{ psi, Tension} \]
\[ \delta_B = 0.020 \text{ in} \]