Assignment - Day 6

1. Finish the problem we started in class ... the one with the horizontal bar 1.5 in. x 1.5 in. x 18 in. long, steel, pinned at one end to a support, and at the other (right) end to a round bar, oriented at 60 deg, 2 in. diameter, 36 in. long, and it pinned at it's other end to a support. The common end is loaded with 10k downward to the right at 45 deg (as sketched in class). Find: a) the deflection of the load point, P (magnitude and direction). Also, find: b) the 'stiffness' of the system with regard to the given load, and, c) the deflection (mag and direction) if the load is increased to 15k (and something bad doesn't happen first).

Strategy:

1) Make some assumptions and conditions ... e.g., let's assume weight and its reactions are small enough to be neglected ... thus we have 2-force members ... 

   Let's also assume the loads are centric ... thus sigma = P/A, delta = PL/AE, and so on ...

2) Solve for the forces in the members using equations in Ch. 2 ...
   delta = PL/AE.

3) Solve the geometry ... compatibility of the deltas of each piece will require a (unique) delta for the point P ...

4) The stiffness of the system ... k = P/delta.

5) Oh, and once we have the k ... if we have a new P (as long as it's in same direction) ... new delta = new P times our k. (And as long as it doesn't break, or something else bad happen, first.)
Assignment 6

Problem 1 ... (finish)  

1.5 x 1.5 Studded

\[ \sin \theta = \frac{18}{L_{BC}} \]

Assume?  

If it doesn't make sense first...  

\[ L_{BC} = 36 \text{ in} \]
Strategy

1. Neglect weight of struts...
2. Solve for forces in members using Statics (Geometry)
3. Solve for first order J's using

\[ F = \frac{P_l}{AE} \]

4. Solve for \( F_0 \) using \( J \)'s...
   Answer, and
5. Discuss...

Solution

Let's look at \( AB \) c

\[ F_1 = F \]

\[ AB \]

\[ F = F_2 \]

\[ Bc \]
Lots of different ways to get the F's...

1. Vector polygon, try
2. Cartesian components
3. CAD
4. MATLAB
5. Complex variables

Let's try vector polygon, try.

\[ \overrightarrow{EF} = 0 \]

\[ \frac{F_2}{\sin 45} = \frac{10}{\sin 60} \]
\[ F_2 = 10^k \frac{\sin 45}{\sin 60} = 8.165 \text{ in} \]

\[ F_1 = \ldots \]
\[ \frac{F_1}{\sin 75} = \frac{10^k}{\sin 60} \quad F_1 = \frac{\sin 75}{\sin 60} \left( 10^k \right) \]

\[ F_1 = 11.15^k \]

Now for the \( \delta' \)'s...

\[ \delta_1 = \frac{P_1 L_1}{A_1 E_1} = \frac{11.15 \times 10^3 \text{ lb} \times (1.8 \text{ in})}{(1.5 \times 1.5) \text{ in}^2} \frac{29 \times 10^6 \text{ lb}}{2.25} \]

\[ \delta_1 = 0.00308 \text{ in elongation} \]

\[ \delta_2 = \frac{P_2 L_2}{A_2 E_2} = \frac{8.165 \times 10^3 \text{ lb} \times (36 \text{ in})}{(2) \frac{1}{4} \text{ in}^2} \frac{29 \times 10^6 \frac{\text{ lb}}{\text{ in}^2}}{3.14} \]

\[ \delta_2 = 0.00323 \text{ in shortening} \]
$S_p$ from first order geometry...
Vectors

\[ 708 \vec{y} + 323 \vec{x} = \text{dir} + d \vec{d} \]

Let's do it...

\[ x \text{dir.} \]

\[ 308 + 0 = -323 + d \cos 30 \]

\[ \frac{308 + 323}{866} = \frac{631}{866} = 0.730 \]

\[ y \text{ dir.} + 1 \]

\[ 0 + y = 323 \cos 30 + d \sin 30 \]

\[ y = 323(0.866) + 730(0.5) = 280 + 364 = 644 \]

\[ 50 \]

\[ 308 \]

\[ 323 \]

\[ 730 \]

\[ 644 \]
\[ \delta P = \sqrt{(308)^2 + (644)^2} \approx 714 \text{ in} \]

\[ \delta P = 0.00714 \text{ in} \]

\[ \beta = \tan^{-1} \frac{308}{644} \quad \therefore \beta = 25.60^\circ \]

\[ K = \frac{P}{\delta P} = \frac{10}{0.00714 \text{ in}} = 1.405 \text{ kips/in} \]

(Note 45)

"Stiffen?"

If we were interested)

Answer: \[ \delta P = 0.007 \text{ in} \]
if something didn't break first--

\[ \sigma_1 = \frac{P_1}{A_1} = \frac{11,150\, \text{lb}}{2.25\, \text{in}^2} = 4,955\, \text{psi}, \text{tension} \]

\[ \sigma_2 = \frac{P_2}{A_2} = \frac{8165\, \text{lb}}{3.14\, \text{in}^2} = 2,600\, \text{psi}, \text{compression} \]

Stresses are not very high, but would probably also look at stability of member BC... since in compression...
could get unstable...
Solution: ... see other sheets.

2. 2.3-6 ... easy ... this is a 'series bar' problem ... just break it up into as many pieces as necessary to identify the P, L, A, and E for each piece (solve for the Ps with statics) ... crunch out the deltas, and add.

3. 2.3-11 ... Solve this problem with 'finite differences' ... by this I mean cut the bar up into 10 segments, each of which you can approximate as being straight (prismatic). Calc the deltas for each piece, add. (In other words, you are 'modeling' the bar as 10 stepped rectangular prismatic pieces instead of one tapered one.) Compare with the 'calculus' solution the author gives. Discuss. (This is a cool problem.)
I can't believe I assigned a problem in the SI system of units... But, here goes...

Strategy... either by inspection or FBDs I'll get the load in each column... then get $f$ for each, and add...

to solve for part b)... I'll simply find out how much more $f$ we need, and then the $P_a$ to get that $f$ in the two columns in series.

**Solution**: By inspection

\[ 400 \text{ kN} \]
\[ A = 3900 \text{ mm}^2 \]
\[ 3.75 \text{ m} \]
\[ 720 \text{ kN} \]
\[ 3.75 \text{ m} \]
\[ A = 15000 \text{ mm}^2 \]
\[ E = 206 \text{ GPa} \]

\[ 400 + 720 = 1120 \text{ kN} \]

\[ \sigma_{aA} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}^2} \]
\[ 3.75 \text{ m} \]
\[ = \frac{1120 \times 10^3 \text{ N/m}^2 \times (10^3)^2 \text{ mm}^2}{206 \times 10^9 \text{ N/m}^2 \times 15000 \text{ mm}^2} \]
\[ \delta = 0.00185 \text{ m} = 1.85 \text{ mm} \]

[Part a]

\[ \delta_{BC} = \frac{400 \times 10^3 \times 3.75 \times 10^6}{206 \times 10^9 \times 39.6} = 1.87 \text{ mm} \]

\[ \delta_{\text{Total}} = 3.72 \text{ mm} \quad \text{answer, part a)} \]

[Part b]

\[ 4.00 - 3.72 = 0.28 \text{ mm} = 0.00028 \text{ m} \]

So...

\[ \delta_{\text{additional}} = \left( \frac{P_{\text{L}}}{AE} \right)_{AB} + \left( \frac{P_{\text{L}}}{AE} \right)_{BC} \]

\[ \delta_{\text{additional}} = P_0 \left( \frac{L_{AB}}{AE_{AB}} + \frac{L_{BC}}{AE_{BC}} \right) = P_0 \left( \text{sum of stiffnesses} \right) \]

\[ \frac{L_{AB}}{A_{\text{AB}} E_{AB}} = \frac{3.75 \text{ m m}^2}{11,000 \text{ m}^2 206 \times 10^3 \times 10^6 \text{ N}} = 1.659 \times 10^{-9} \text{ m} \]

\[ \frac{L_{BC}}{A_{\text{BC}} E_{BC}} = \frac{3.75}{3960 \times 206 \times 10^3 \text{ N}} = 4.668 \times 10^{-9} \frac{\text{m}}{\text{N}} \]
Total flexibility:

\[ 1.655 \times 10^{-9} + 4.668 \times 10^{-9} = \]

\[ 6.322 \times 10^{-9} \, \frac{m}{N} \]

\[ 6.322 \times 10^{-9} \, \frac{m}{N} \times \frac{1000 \, mm}{n} = 6.322 \times 10^{-6} \, \frac{mm}{N} \]

Okay, so now:

\[ S = 0.00028 \, m = 0.28 \, mm = P_0 \, 6.322 \times 10^{-6} \, \frac{mm}{N} \]

\[ P_0 = \frac{0.28 \, mm}{6.322 \times 10^{-6} \, mm} = 44,360 \, N \]

\[ P_0 = 44 \, kN \]

Answer: a) 3.72 mm
b) 44 kN

Discussion: We could check, but since my answers agree with those in the back of the book, I'll opt out.

Going through the effort of casting the problem in terms of "flexibilities" may not have been worth much, but I did it anyway.
Problem 2.3-11 ...

Strategy: break the bar into pieces for which we'll calc the PL/AE for each piece, and add ...

... the height of the bar changes ... at the rate of 6-4 = 2 in. ... per 5 ft or 60 in ... = 0.033 in./in.

Break bar into 10 pieces, calc the height at the beginning, end, and midpoint of each piece, and to the PL/AE based on the height at the midpoint.

Each piece has a common P, L, and E ...

\[ \text{PL/E} = \frac{25,000 \text{ lb (6 in.)}}{30,000,000 \text{ lb/in.}^2} = 0.005 \text{ in.}^3 \]

Each piece will have an area of mid-height x thickness (1.0 in.)

<table>
<thead>
<tr>
<th>Piece #</th>
<th>Start height</th>
<th>End height</th>
<th>Mid-height</th>
<th>Area</th>
<th>(1/A)(PL/E)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in. 2</td>
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<tr>
<td>1</td>
<td>4</td>
<td>4.2</td>
<td>4.1</td>
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<td>5.9</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

Total ... \[0.0101 \text{ in.}\]
Compare to `exact- formula ...

\[ \Delta = \frac{PL}{(Et(b_2-b_1))} \times \ln \left( \frac{b_2}{b_1} \right) = \]

\[ = \frac{(25,000 \text{ lb} \times 60 \text{ in.})}{(30,000,000 \text{ lb/in.}^2 \times 1 \text{ in.}(6-4 \text{ in.}))} \times \ln \left( \frac{6}{4} \right) = \]

**Answer** = 0.0101 in. ...

**Discussion:** ... whoa – that's wild .. accurate to 3 sig figs in this case.