1. Using the parallel-axis theorem, determine, for the shaded area shown, with respect to the centroidal $x$ and $y$ axes, the product of inertia AND EITHER the moment of inertia with respect to the $x$ axis OR the moment of inertia with respect to the $y$ axis.

![Diagram of a shaded area with dimensions and formulas for moments of inertia.]

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>( I_x = \frac{1}{12} bh^3 )</th>
<th>( I_y = \frac{1}{12} b^2 h )</th>
<th>( I_z = \frac{1}{2} bh^2 )</th>
<th>( I_y' = \frac{1}{3} bh^2 )</th>
<th>( I_c = \frac{1}{2} bh(b^2 + h^2) )</th>
</tr>
</thead>
</table>

\[ I_{xy} = I_{x'y'} + xy A \]
2. Two 10° wedges of negligible weight are used to move and position the 400-lb block. Knowing that \( \mu_s = 0.25 \) and \( \mu_k = 0.20 \) at all surfaces of contact, determine the smallest force \( P \) that should be applied as shown to the bottom wedge.

\[
F = \mu N \\
\tan \phi = \mu
\]
3. Determine by direct integration the moment of inertia of the shaded area with respect to EITHER the $x$ axis OR the $y$ axis.

\[ dA = (a - x) dy \]
\[ dI_x = y^2 dA \]
\[ I_x = \int y^2 dA \]

\[ dA = y dx \]
\[ dI_y = x^2 dA \]
\[ I_y = \int x^2 dA \]
First find centroid - THIS WAS GIVEN FOR THE EXAM

<table>
<thead>
<tr>
<th>Shape</th>
<th>( x )</th>
<th>( y )</th>
<th>( A )</th>
<th>( \bar{x}A )</th>
<th>( \bar{y}A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.65</td>
<td>4.8</td>
<td>1.3</td>
<td>0.845</td>
<td>6.24</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>2.4</td>
<td>1.9</td>
<td>0.475</td>
<td>4.56</td>
</tr>
<tr>
<td>3</td>
<td>1.80</td>
<td>0.25</td>
<td>1.8</td>
<td>3.24</td>
<td>0.45</td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum xA}{\sum A} = \frac{4.56}{5.0} = 0.912''
\]

\[
\bar{y} = \frac{\sum yA}{\sum A} = \frac{11.25}{5.0} = 2.25''
\]

Location of each rectangle's centroid relative to composite centroid:

- Distance: 0.912 - 0.65 = 0.262''
- Distance: 0.912 - 0.25 = 0.662''
- Distance: 1.80 - 0.912 = 0.888''

Distance: 0.5 + 3.8 + 0.65 - 2.25 = 2.55''

Distance: 0.5 + 1.9 - 2.25 = 0.15''
\[ \overline{I} = \overline{I} + \overline{A}d^2 \]

\[ \overline{I} \] is the centroidal \[ \overline{I} \] that you are moving to another parallel axis. In this problem, \[ \overline{I} \] is about \( C_1, C_2, \& C_3 \) and we are moving to \( C_7 \) (which is the centroid of the figure we are moving any of the rectangles we are moving.

\[ I_{x_1}' = \frac{1}{12} (13)(1)^3 + (13)(1)(2.55^2) = 0.1063 + 8.4533 = 8.5616 \]

\[ I_{x_2}' = \frac{1}{12} (0.5)(3.8^3) + (0.5)(3.8)(0.15^2) = 2.2863 + 0.1428 = 2.4291 \]

\[ I_{x_3}' = \frac{1}{12} (3.6)(0.5^3) + (3.6)(0.5)(2.00^2) = 0.0375 + 7.2000 = 7.2375 \]

\[ I_{x_1} = 8.5616 + 2.4291 + 7.2375 = 18.2222 \text{ in}^4 \]

Report 18.13 in\(^4\) \( \checkmark \)

\[ I_{y_1}' = \frac{1}{12} (1.5^3)(1) + (1.5)(1)(0.262^2) = 0.1831 + 0.0892 = 0.2723 \]

\[ I_{y_2}' = \frac{1}{12} (0.5^3)(3.8) + (0.5)(3.8)(0.462^2) = 0.0836 + 0.8327 = 0.9163 \]

\[ I_{y_3}' = \frac{1}{12} (3.6^3)(0.5) + (3.6)(0.5)(0.888^2) = 1.9440 + 1.4194 = 3.3634 \]

\[ I_{y_1} = 0.2723 + 0.8723 + 3.3634 = 4.5100 \text{ in}^4 \]

Report 4.51 in\(^4\) \( \checkmark \)
In each of the three rectangles for the $x'y'$ axes through the centroid, $I_{x'y'} = 0$

$I_{x'y'}^1 = 0 + (-0.262)(2.55)(1.3)(1.6) = -0.8685$

$I_{x'y'}^2 = 0 + (-0.662)(0.15)(0.5)(3.8) = -0.1887$

$I_{x'y'}^3 = 0 + (0.888)(-2.00)(3.6)(0.5) = -3.1968$

$I_{x'y'}^{TOTAL} = -4.25 \text{ in}^4$
\[
\begin{align*}
\mu_5 &= 0.85 \\
\tan \phi_3 &= \mu_5 = 0.85 \\
\phi_3 &= 14.04^\circ \\
10^\circ & \\
14.04^\circ & \\
N_2 & \\
400^\circ & \\
14.04^\circ & \\
N_1 & \\
R_1 & \\
10^\circ & \\
R_2 & \\
14.04^\circ & \\
R_3 & \\
P & \\
14.04^\circ & \\
R_3 & \\
492.98^\circ & \\
-24.04 & \\
75.96^\circ & \\
-14.04 & \\
90 & \\
75.96^\circ & \\
65.96 & \\
24.04 & \\
51.92^\circ & \\
104.04^\circ & \\
24.04 & \\
90 & \\
-14.04 & \\
75.96^\circ & \\
65.96 & \\
38.08^\circ & \\
158.08^\circ & \\
14.04+24.04 & \\
38.08^\circ & \\
R_2 &= 492.98^\circ \\
\frac{R_2}{\sin 14.04^\circ} &= \frac{400}{\sin 51.92^\circ} \\
R_2 &= 492.98 \\
P &= 492.98^\circ \\
\frac{P}{\sin 38.08^\circ} &= \frac{492.98}{\sin 75.96^\circ} \\
P &= 313.41 \\
\end{align*}
\]
\( I_y = \int x^2 \, dA = \int x^2 \, dx \)

\[ I_y = \int_0^2 x^2 \, \frac{a^2}{x} \, dx = a^2 \int_0^2 x \, dx \]

\[ I_y = a^2 \left[ \frac{x^2}{2} \right]_0^2 = a^2 \left[ \frac{4a^2}{2} - \frac{a^2}{2} \right] = 2^2 \frac{3a^2}{2} \]

\[ I_y = \frac{3a^4}{2} \]
This area is not really amenable to creating an element parallel to the \( x \) axis and integrating over definite limits of \( dy \). So use a vertical element as in text Figure 9.5, recognizing that \( dx = \frac{1}{3}y^3 dy \).

\[
\begin{align*}
\int x &= \frac{1}{3} \int y^3 dy = \frac{1}{3} \int (\frac{2}{x})^3 dx = \frac{8}{3} \int \frac{1}{x^3} dx \\
&= \frac{8}{3} \left[ -\frac{1}{2} \right] = -\frac{4}{3} \left[ \frac{1}{x^2} \right]_a^b \\
&= -\frac{a^6}{6} \left[ 1 - \frac{1}{4a^2} \right] = -\frac{a^6}{6} \left[ \frac{1}{4a^2} - \frac{1}{4a^2} \right] \\
&= -\frac{a^6}{6} \left[ -\frac{3}{4a^2} \right] = \frac{a^6}{8a^2} = \frac{a^4}{8}
\end{align*}
\]